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Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2020/2021

| | | |
|---|---|---|
| COURSE NAME | : | ENGINEERING TECHNOLOGY MATHEMATICS III |
| COURSE CODE | : | BDU 21103 |
| PROGRAMME CODE | : | BDC / BDM |
| EXAMINATION DATE | : | JULY 2021 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B** |

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES



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## PART A

**Q1** (a) Given that $y' + 2y = xe^{3x}$ with initial condition $y(0) = 0$.

(i) Calculate the approximation of the solution for $x = 0(0.2)1$ by using classical fourth-order Runge-Kutta (RK4) method.

(8 marks)

(ii) The exact solution for **Q1(a)(i)** is given by

$$y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}.$$

Hence, find its error.

(2 marks)

(b) Given the matrix

$$A = \begin{pmatrix} 1 & 1.5 & -1 \\ 2 & 6 & -4 \\ -6 & 1 & 3 \end{pmatrix}.$$

Identify the smallest (in absolute value) eigenvalue and its corresponding eigenvector by using inverse power method. Use $\boldsymbol{v}^{(0)} = (1 \quad 1 \quad 1)^T$ and do your calculation until $|m_{k+1} = m_k < 0.005|$.

(10 marks)

**Q2** (a) The temperature distribution $u(x,t)$ is a 5m long metal rod is governed by the problem

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}, \quad (0 < x < 5, \ t > 0)$$

with the conditions

$$u(0,t) = u(5,t) = 0, (t > 0)$$

and

$$u(x,0) = x^2(5-x).$$

Analyze the solution for $u(x, 0.1)$ by using implicit finite difference method (Crank-Nicolson method) if $\Delta x = h = 1.25$ and $\Delta t = k = 0.1$.

(13 marks)

(b) Determine $a, b, c, d, e$ and $f$ in the **Table Q2(b)** by using the Newton's divided-difference method.

(7 marks)

**PART B**

**Q3** (a) Integrate the given integrals over three dimension Cartesian coordinate

$$\int_{-1}^{2}\int_{1}^{z^2}\int_{0}^{y+z} 3yz^2\ dxdydz.$$

(10 marks)

(b) Calculate the volume of the solid that lies inside cone $z = 1 - \sqrt{x^2 + y^2}$ and above the plane $z = -1$ by using cylindrical coordinates.

(10 marks)

**Q4** (a) Given nonlinear equations $g(x) = e^x + 2^{-x}$ and $h(x) = 6 - 2\cos x$. Analyze the root of $g(x) - h(x) = 0$ in the interval [1,2] by using bisection method. Iterate until $|f(x_i)| < 0.005$.

(10 marks)

(b) Given

$$z = e^{xy}, \quad x = u + v, \quad y = \frac{u}{v}.$$

Differentiate $z$ partially and identify $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

(10 marks)

**Q5** (a) Outline the local extreme and saddle point (if exists) for the function

$$f(x,y) = 2x^2 - y^3 - 2xy + 4.$$

(10 marks)

(b) Analyze the system of linear equations by using Thomas algorithm method.

$$\begin{pmatrix} 0 & 0 & 3 & 4 \\ 2 & 9 & 1 & 0 \\ 0 & 1 & 9 & 4 \\ 2 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 8 \\ 9 \end{pmatrix}$$

(10 marks)

**Q6** (a) The upward velocity of a rocket, measured at 3 different times, is shown in **Table Q6.**The velocity over the time interval $5 \le t \le 12$ is approximated by a quadratic expression as

$$v(t) = a_1 t^2 + a_2 t + a_3 .$$

Determine the values of $a_1, a_2$ and $a_3$ by using Gauss – Elimination method.

(10 marks)

(b) A cube is defined by three inequalities $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. The cube has a density function $\delta(x, y, z) = k(x^2 + y^2 + z^2)$. Given that the mass of the cube is $k$. Outline its center of gravity.

(10 marks)

-END OF QUESTIONS –

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**Table Q2(b)**: Newton's divided difference method.

| | | | | |
|---|---|---|---|---|
| $x_0 = 1.0$ | $f[x_0] = a$ | $f[x_0, x_1] = 0.84$ | $f[x_0, x_1, x_2] = f$ | $f[x_0, x_1, x_2, x_3] = 0.096$ |
| $x_1 = 1.4$ | $f[x_1] = b$ | $f[x_1, x_2] = d$ | $f[x_1, x_2, x_3] = -0.187$ | |
| $x_2 = 1.6$ | $f[x_2] = c$ | $f[x_2, x_3] = e$ | | |
| $x_3 = 2.0$ | $f[x_3] = 0.693$ | | | |

**Table Q6**: Upward velocity of a rocket.

| Time , $t$ (seconds) | Velocity, $v$ (meters/second) |
|---|---|
| 5 | 106.8 |
| 8 | 177.2 |
| 12 | 279.2 |

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## Formulas

### Partial differential equations

Heat equation: Implicit Crank-Nicolson method:

$$\frac{\partial}{\partial t}u\left(x_i, t_{j+\frac{1}{2}}\right) = c^2\frac{\partial^2}{\partial x^2}u\left(x_i, t_{j+\frac{1}{2}}\right)$$

$$\frac{u_{i,j+1}-u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{h^2}+\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}\right)$$

### Interpolation

Newton divided-difference method

$$P_n(x) = f_0^{[0]} + f_1^{[0]}(x-x_0) + f_2^{[0]}(x-x_0)(x-x_1) + \cdots + f_n^{[0]}(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

### System of linear equations

Thomas Algorithm:

| $i$ | 1 | 2 | ... | $n$ |
|---|---|---|---|---|
| $d_i$ | | | | |
| $e_i$ | | | | |
| $c_i$ | | | | |
| $b_i$ | | | | |
| $\alpha_1 = d_1$<br>$\alpha_i = d_i - c_i\beta_{i-1}$ | | | | |
| $\beta_i = \frac{e_i}{\alpha_i}$ | | | | |
| $y_1 = \frac{b_1}{\alpha_1}$<br>$y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$ | | | | |
| $x_n = y_n$<br>$x_i = y_i - \beta_i x_{i+1}$ | | | | |

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**Eigenvalue**

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,.....$

Inverse Power Method: $\lambda_{\text{smallest}} = \dfrac{1}{\lambda_{\text{Shifted}}}$

**Cylindrical coordinates:**

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \quad x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$$

$$V = \iiint_G dV = \iiint_G dz\, r\, dr\, d\theta$$

**Fourth-order Runge-Kutta Method**

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$
$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right),$$
$$k_4 = hf(x_i + h, y_i + k_3).$$

**Mass**

$$m = \iiint_G \delta(x,y,z)\, dV$$

**Center of Gravity $(\bar{x}, \bar{y}, \bar{z})$**

$$\bar{x} = \frac{1}{m}\iiint_G x\,\delta(x,y,z)\, dV, \bar{y} = \frac{1}{m}\iiint_G y\,\delta(x,y,z)\, dV, \bar{z} = \frac{1}{m}\iiint_G z\,\delta(x,y,z)\, dV.$$