UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2020/2021

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDX 10502
PROGRAMME CODE	:	BDX
EXAMINATION DATE	:	JULY 2021
DURATION	:	2 HOURS
INSTRUCTION QUESTIONS	:	ANSWER **FOUR (4)** FROM FIVE (5) QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Evaluate the Lagrange Interpolating polynomial for data f (0) = 1 , f (2) = -1, f (4) = -1 and f (6) = 1. Hence, evaluate f (3), f (5) and f (6.5) , if applicable.

(25 marks)

Q2 (a) Convert to spherical coordinates and solve

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2 + y^2 + z^2)dzdydx.$$

(12 marks)

(b) Solve the volume of the solid G, that lies between the two cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ bounded above by paraboloid z = 12 - $x^2 - y^2$ and below by xy-plane.

(13 marks)

Q3 (a) A periodic function f (x) is defined by

$$f(x) = x, \qquad -1 < x < 1$$

and

$$f(x) = f(x+2)$$

i) Sketch the graph of the function over -3 < x < 3

(6 marks)

ii) Evaluate the Fourier coefficients corresponding to the function.

(10 Marks)

iii) Solve the corresponding Fourier series.

(9 Marks)

Q4 Solve the system of linear equations below by using Thomas method

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 5 & 0 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 10 & 11 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{Bmatrix}$$

(25 marks)

Q5 Given a system of response following the function $y = x^6 - x - 1$. Solve the stable condition (zero) of the system in the range [1,2] using Bisection method. Iterate until the tolerance error $(b - a)/2 \leq 0.005$

(25 marks)

-END OF QUESTION –

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FORMULA

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi d\rho d\phi d\theta$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

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Centre of Mass
Non-Homogeneous Lamina:

$$(\bar{x},\bar{y})=\left(\frac{M_y}{m},\frac{M_x}{m}\right)$$

Centroid
Homogeneous Lamina:

$$\bar{x}=\frac{1}{Area\ of\quad R}\iint_R x\,dA \text{ and } \bar{y}=\frac{1}{Area\ of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y=\iint_R x^2\delta(x,y)dA$

b. $I_x=\iint_R y^2\delta(x,y)dA$

c. $I_o=\iint_R (x^2+y^2)\delta(x,y)dA$

In 3-D: Solid
Given that $\delta(x,y,z)$ is a density of solid

Mass, $m=\iiint_G \delta(x,y,z)dV$

If $\delta(x,y,z)=c$, where c is a constant, $m=\iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz}=\iiint_G x\delta(x,y,z)dV$

b. About *xz*-plane, $M_{xz}=\iiint_G y\delta(x,y,z)dV$

c. About *xy*-plane, $M_{xy}=\iiint_G z\delta(x,y,z)dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z})=\left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla\times\mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$

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Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)
b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)
c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)
d. If $D = 0$
   The test is inconclusive

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$