UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2020/2021

COURSE NAME	:	DIFFERENTIAL EQUATIONS
COURSE CODE	:	BDA 24303
PROGRAMME	:	BDD
EXAMINATION DATE	:	JULY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ONE (1)** QUESTION **ONLY.** PART B: ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF EIGHT **(8)** PAGES

PART A: ANSWER ONE (1) QUESTION ONLY.

Q1 A rod of length π is fully insulated along it sides. Assuming the initial temperature is defined by,

$$f(x)=\begin{cases} x, & 0<x<\dfrac{\pi}{2} \\ \pi-x, & \dfrac{\pi}{2}<x<\pi \end{cases}$$

and at $t=0$, the ends are dipped into cold water and held at temperature of 0^{o}C. The heat equation given as,

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

(a) Analyze the given partial differential equation by using the method of separation of variables and prove that the solution of the heat transfer problem above gives as,

$$u(x,t)=\sum_{n=1} A_n \sin nx\, e^{-n^2 t}$$

where A_n are an arbitrary constant.

(15 marks)

(b) Solve the particular solution for the general solution obtained from the **Q1 (a).**

(5 marks)

Q2 A rod of length 2 m is fully insulated along its sides. The temperature at x is initially $100\sin\left(\frac{\pi x}{2}\right)$ °C, and at $t = 0$, the ends are dipped into cold water and held at temperature of 0°C. If the heat equation given as

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

(a) By using the method of separation of variables, discover the expression for the temperature at any point at a distance x from one end at any subsequent time t second after $t = 0$.

(15 marks)

(b) Solve the particular solution for the general solution obtained from the **Q2 (a).**

(5 marks)

PART B: ANSWER ALL QUESTIONS

Q3 A half-range expansions given as the following function:

$$f(x) = \frac{\pi}{2} - x \quad \text{for} \quad 0 < x < \pi$$

(a) Sketch a graph of $f(x)$ in the interval $0 < x < \pi$.

(2 marks)

(b) Solve the given half-range expansion of the function as an ***even function*** and sketch the periodic extension for the series obtained for $-2\pi < x < 2\pi$.

(10 marks)

(c) Solve the given half-range expansion of the function as an ***odd function*** and sketch the periodic extension for the series obtained for $-2\pi < x < 2\pi$.

(8 marks)

Q4 Obtain the particular solution for:

$$2y'' + 2y = 2\sin x \qquad y(0) = 0,\ y'(0) = 1$$

(a) By using Laplace transform method.

(10 marks)

(b) By using method of variation parameter.

(10 marks)

Q5 (a) A population of a village grows proportion to its current population. The initial population is 10,000 and grows 9% per year. This can be modeled

$$\frac{dP}{dt} = 0.09\,P$$

Determine

(i) the equation to model the population.
(ii) the population after 5 years.
(iii) how long it will take the population to double.

(10 marks)

(b) A forced system is given by:

$$my'' + ny' + ky = 10\cos(2x)$$

Solve for the steady state solution in the case where $m = 1$, $n = 3$, and $k = 2$.

(10 marks)

Q6 A periodic function $f(x)$ is defined as the following function:

$$f(x)=\begin{cases} 0 & -\pi < x < -\dfrac{\pi}{2} \\ 1 & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ 0 & \dfrac{\pi}{2} < x < \pi \end{cases}$$

and

$$f(x)=f(x+2\pi)$$

(a) Sketch a graph of this function over the interval $-3\pi < x < 3\pi$.

(3 marks)

(b) Solve the given the function and show that its Fourier series is given by,

$$f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(2n-1)x}{2n-1}$$

(12 marks)

(c) Using the results obtained in **Q3(b)** and by setting an appropriate value of x, conclude that,

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots-\ldots$$

(5 marks)

- END OF QUESTION -

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

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Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where;

$u_1=-\int \frac{y_2 g(x)}{W}dx$ and $u_2=\int \frac{y_1 g(x)}{W}dx$ $W(y_1,y_2)=\begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}=y_1y'_2-y_2y'_1$

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s)-y(0)$
$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$