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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2020/2021

COURSE NAME	:	CALCULUS FOR ENGINEERS
COURSE CODE	:	BDA 14403
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ALL** QUESTIONS PART B: ANSWER **TWO (2)** FROM **THREE (3)** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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PART A:

Q1 (a) Given the force field $\mathbf{F} = (z^3 \cos x + 2xy^2)\mathbf{i} + (2x^2 y - 2)\mathbf{j} + (3z^2 \sin x - 4)\mathbf{k}$

(i) Prove that **F** is conservative (4 marks)

(ii) By using formula $\nabla\emptyset = \mathbf{F}$, find a scalar potential $\emptyset$ for **F**. (4 marks)

(iii) Hence, compute the amount of work done against the force field F in moving an object from the point (0, -1, 1) to $(\frac{1}{2}\pi, 2, 2)$. (2 marks)

(b) Evaluate $\oint_C \mathbf{F} \bullet d\boldsymbol{r}$ by using Stoke's Theorem, where $\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ and C is perimeter of closed triangle of the plane $x + 2y + z = Q$ in the first octant in the counterclockwise direction, when viewed from the positive z-axis.

Q – The last digit of your matrix number
For example, a student with the matrix number CD200079 will have the values of Q = 9. Use 1 if the last digit is of your matrix number is 0.

(10 marks)

Q2 (a) Given $\mathbf{F}(x,y,z) = x^4y^2\,\mathbf{i} + y^2z\,\mathbf{j} + xz^3\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$. Find the integrand that derived when using Gauss's Theorem to evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\ dS$. **(Do not solve the integration)**

(4 marks)

(b) Solve the integral $\int_C (3x - y)dx + (2x + 3y)dy$ along the curve $x = y^3$ from $(0,0)$ to (1, 1).

(8 marks)

(c) Use Green's Theorem to evaluate $\int_C 3xy\,dx + 2xy\,dy$, where C is a counterclockwise oriented rectangle, bounded by $x = -2$, $x = 4$, $y = 1$ and $y = 2$.

(8 marks)

Q3 (a) Calculate the work done in moving a particle in the vector field:

$$\mathbf{F} = xz\,\mathbf{i} + xy^2\mathbf{j} + 3xz\mathbf{k}$$

along the intersection of the plane $x + z = M$ and the cylinder $x^2 + y^2 = M + 2$, in the counterclockwise direction.

M– The last digit of your matrix number

For example, a student with the matrix number CD200079 will have the values of M = 9. Use 1 if the last digit is of your matrix number is 0.

(10 marks)

(b) Analyze all relative extrema, relative minima, and saddle points (if any) of $f(x, y) = xy - x^3 - y^2$.

(10 marks)

PART B:

Q4 (a) Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y}$ if it exist. If not, show that the limit does not exist.

(5 marks)

(b) Given $z = x^2(1 + y)^3$, the value of (x,y) changes from (2,9) to (2.03,8.9). Use the total differential to find the approximate value and also exact value for z.

(7 marks)

(c) A lamina enclosed between the circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and x=0 (upper half of annulus) has density function $\delta(x, y) = yx^2$. Find its mass.

(8 marks)

Q5 (a) Find the equation of tagent planes to the surface $f(x, y) = Kx^2 + 5xy - y^2$ at (1,-1, 1).

K– The last digit of your identification card

For example, a student with the identification card 951002-01-5762 will have the values of $K = 2$. Use 1 if the last digit is 0.

(5 marks)

(b) By using double integrals, find the surface area of the portion of hemisphere $z = \sqrt{25 - x^2 - y^2}$ that lies above the region R by the disc $x^2 + y^2 \leq 9$

(6 marks)

(c) Find the centroid of the solid G bounded above sphere $x^2 + y^2 + z^2 = 4$ and below by cone $z = \sqrt{x^2 + y^2}$. Assume that the solid has constant density is equal to 1.

(9 marks)

Q6 (a) Given the function $f(x,y) = -y\sqrt{9 - x^2 - y^2 - z^2}$. Find the domain and range of the function. Then sketch the domain.

(5 marks)

(b) Sand is taken out from its container at the rate of 104 cm^3/min. By the same pace, the sand is then put in vertical cone with increasing rate of radius 2 cm/min. If when 30 cm^3 of sand was taken out , the radius of the cone is 6 cm. Find the increasing rate of the height of the sand at that time.

(7 marks)

(c) Find the coordinate $\bar{y}$ and $\bar{z}$ for the centroid of solid enclosed by surface $z = y^2$ and planes $x = 0, x = 1$ and $z = 1$. Assume that the solid has constant density is equal to 1.

(8 marks)

-END OF QUESTIONS –

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FORMULA

Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
 $f(x,y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
 $f(x,y)$ has a local minimum value at (a,b)

c. If $D < 0$
 $f(x,y)$ has a saddle point at (a,b)

d. If $D = 0$
 The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi d\rho d\phi d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

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Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y})=\left(\frac{M_y}{m},\frac{M_x}{m}\right)$$

Centroid

Homogeneous Lamina:

$$\bar{x}=\frac{1}{Area\,of\quad R}\iint_R x\,dA \text{ and } \bar{y}=\frac{1}{Area\,of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y=\iint_R x^2\delta(x,y)dA$

b. $I_x=\iint_R y^2\delta(x,y)dA$

c. $I_o=\iint_R (x^2+y^2)\delta(x,y)dA$

In 3-D: Solid

Given that $\delta(x,y,z)$ is a density of solid

Mass, $m=\iiint_G \delta(x,y,z)dV$

If $\delta(x,y,z)=c$, where c is a constant, $m=\iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz}=\iiint_G x\delta(x,y,z)dV$

b. About *xz*-plane, $M_{xz}=\iiint_G y\delta(x,y,z)dV$

c. About *xy*-plane, $M_{xy}=\iiint_G z\delta(x,y,z)dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z})=\left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint\limits_G (y^2+z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint\limits_G (x^2+z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint\limits_G (x^2+y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla\times\mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t)=\dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t)=\dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector**, $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$

Curvature

$$\kappa=\frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho=\frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx+N dy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}dS=\iiint_G \nabla\cdot\mathbf{F}dV$$

Stoke's Theorem

$$\oint_C \mathbf{F}\cdot dr=\iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}dS$$

Arc Length

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}, t\in[a,b]$, hence, the **arc length**,

$$s=\int_a^b\|\mathbf{r}'(t)\|\,dt=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$