Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAM 21303 / DAE 23403
PROGRAMME CODE	:	DAM / DAE
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Solve the second order differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=0$ given that when $x=0$, $y=2$ and $\frac{dy}{dx}=1$.

(7 marks)

(b) Find the general solution of the second order differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^{2x}+3$ by using method of variation of parameters

(13 marks)

Q2 (a) Solve the following integral by using integration by parts:

$$\int e^{-3t}\cos 2t\ dt.$$

(8 marks)

(b) Evaluate the following integrals.

(i) $\int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx.$

(4 marks)

(ii) $\int \frac{x^2+1}{\sqrt{x}}+\sec^2 5x\ dx.$

(2 marks)

(c) Solve the following integral by using integration by partial fractions:

$$\int \frac{x+7}{x^2(x+2)}\,dx.$$

(6 marks)

Q3 (a) **Figure Q3 (a)** shows the region **W** bounded by curves $y^2=10-x$ and $x=(y-2)^2$.

(i) Find the coordinates of **A** and **B**.

(5 marks)

(ii) Evaluate the area of the region **W**.

(5 marks)

(b) Consider region **R** is enclosed by $y = 4 - x^2$ and $y = x^2$.

(i) Sketch the graph and identify the region **R**.

(3 marks)

(ii) Find the volume of the solid generated when the region **R** is revolved about $y-$axis.

(7 marks)

Q4. (a) Given the first order differential equation:

$$\left(y^2 - x^2\right)dy - 2xydx = 0.$$

(i) Show that the equation is a homogeneous equation

(2 marks)

(ii) Find the general solution of the equation

(8 marks)

(b) Given the first order linear differential equation:

$$x\frac{dy}{dx} + 3y = e^{-2x}.$$

(i) State two reasons why the differential equation is linear.

(2 marks)

(ii) Thus, solve the equation.

(8 marks)

Q5 (a) As a practical student in IWC company, you have been given a task to remove a heavy metal with its core temperature of $500°F$ from a furnace and placed the metal on a table in a room that had a constant temperature of $45°F$. One hour after it is removed the core temperature is $150°F$. The temperature of the metal must be below $50°F$ before you can transfer it to the next section.

(i) Given $\dfrac{dT}{T - T_s} = -kdt$. Show that $T - T_s = Ae^{-kt}$.

(4 marks)

(ii) By using $T - T_s = Ae^{-kt}$, with $T(0) = 500^\circ\text{F}$ and $T_s = 45^\circ\text{F}$, find the constant A. Hence find $T(t)$.

(4 marks)

(iii) Given the observed temperatures of the metal, given $T(1)=150^\circ\text{F}$, find the constant k.

(4 marks)

(iv) Find the time taken for the temperature of the metal to be below 50°F.

(3 marks)

(b) The world population growth is described by $y(t)=y_0e^{k(t-t_0)}$ with t measured in years.

(i) If the population increased 2019 by 2.5% from 2018 to 2019, find k.

(3 marks)

(ii) If the population in $t_0=2018$ was 32 million people, find the actual population for 2021 predicted by the given equation.

(2 marks)

- END OF QUESTIONS -

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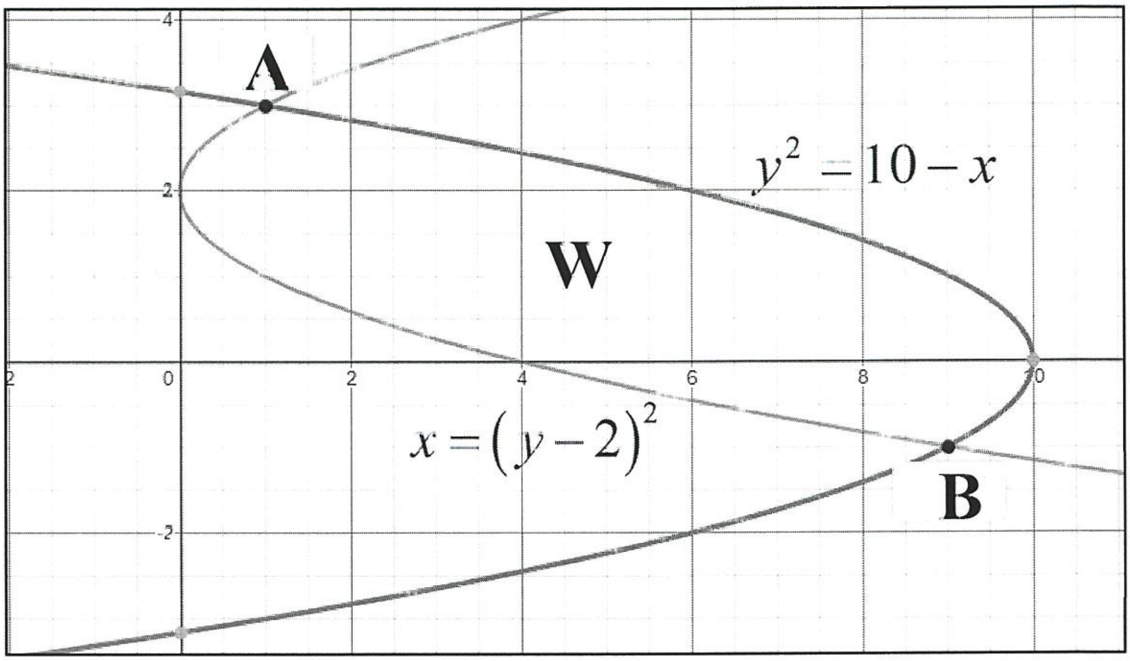

Figure Q3 (a)

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Formula

Table 1: Characteristic Equation and General Solution

Homogeneous Differential Equation: $ay'' + by' + cy = 0$

Characteristics Equation: $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of Characteristics Equation	General Solution
1	Real and Distinct: $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	Real and Equal: $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	Complex Roots: $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2: Particular Solution of Nonhomogeneous Equation

$ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)$
Ce^{ax}	$x^r\left(Pe^{ax}\right)$
$C\cos\beta x$ or $C\sin\beta x$	$x^r\left(P\cos\beta x + Q\sin\beta x\right)$
$P_n(x)e^{ax}$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)e^{ax}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$ or	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\cos\beta x$ + $x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\sin\beta x$

Notes: r is the smallest non negative integer to ensure no alike term between $y_p(x)$ and $y_h(x)$.

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Table 3: Variation of Parameters Method

Homogeneous solution, $y_h(x) = Ay_1 + By_2$	
Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$	
$u_1 = -\int \frac{y_2 f(x)}{aW}\,dx + A$	$u_2 = \int \frac{y_1 f(x)}{aW}\,dx + B$
General solution, $y(x) = u_1y_1 + u_2y_2$	

Table 4: Trigonometry Identities

$\sin^2 t + \cos^2 t = 1$
$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$
$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$

Table 5: Partial Fraction

$\frac{a}{(s+b)(s-c)} = \frac{A}{s+b} + \frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$
$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$

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Table 6: Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{dx} x^n = nx^{n-1}$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\frac{d}{dx} \ln x = \frac{1}{x}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b} \ln\lvert a-bx\rvert + C$	$\frac{d}{dx} \ln(ax+b) = \frac{a}{ax+b}$
$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$	$\frac{d}{dx} e^{ax} = ae^{ax}n$
$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$	$\frac{d}{dx} \sin ax = a \cos ax$
$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$	$\frac{d}{dx} \cos ax = -a \sin ax$
$\int \sec^2 x \, dx = \tan x + C$	$\frac{d}{dx} \tan x = \sec^2 x$
$\int \csc^2 x \, dx = -\cot x + C$	$\frac{d}{dx} \cot x = -\csc^2 x$
$\int u \, dv = uv - \int v du$	$\frac{d}{ds}(uv) = u\frac{dv}{ds} + v\frac{du}{ds}$
$\int_a^b f(x) dx = F(b) - F(a)$	$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$

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Area of Region

$$A = \int_a^b \left[f(x) - g(x) \right] dx \qquad \text{or} \qquad A = \int_c^d \left[w(y) - v(y) \right] dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x \, f(x) \, dx \qquad \text{or} \qquad V = \int_c^d 2\pi y \, f(y) \, dy$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx \qquad \text{or} \qquad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dy$$

Simpson's Rule

$$\int_a^b f(x) \, dx \approx \frac{h}{3} \left[\left(f(a) + f(b) \right) + 4 \sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a + ih) + 2 \sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(a + ih) \right] ; \qquad n = \frac{b - a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x) \, dx \approx \frac{h}{2} \left[f(a) + f(b) + 2 \sum_{i=1}^{n-1} f(a + ih) \right] ; \qquad n = \frac{b \quad a}{h}$$