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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	RISK THEORY
COURSE CODE	:	BWA 40803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

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THIS QUESTION PAPER CONSISTS OF **SEVEN(7)** PAGES

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Q1 (a) The owner of an automobile insures its automobile against damage by purchasing an insurance policy with a deductible of RM450. In the event that the automobile is damaged, repair costs can be modelled by a uniform random variable on the cost interval $(0, 2250)$. Determine the standard deviation of the insurance payment in the event that the automobile is damaged.

(17 marks)

(b) Suppose that a person maximizes his expected utility with the utility function given by $u(w) = \sqrt{w}$. Suppose that the person engages in a risky venture which leaves him with either RM144 or RM36 with equal probability.

(i) What is the certainty equivalent of this business venture?

(ii) What is the risk premium?

(8 marks)

Q2 (a) A general insurance company has a portfolio of fire insurance policies, which offer cover for just one fire each year. Within the portfolio, there are three types of buildings for which the average cost of a claim and probability of a claim are given in **Table Q2(a)**:

Table Q2(a): Portfolio of Fire Insurance Policies

Type of building	*Number of Risks Covered*	*Average Cost of a Claim*	*Probability of a Claim*
Small	147	12.4	0.031
Medium	218	27.8	0.028
Large	21	130.3	0.017

Assume that the cost of a claim has an exponential distribution, and that all the buildings in the portfolio represent independent risks for this insurance cover.

(i) Calculate the mean and standard deviation of annual aggregate claims from this portfolio of insurance policies.

(ii) Using a normal distribution to approximate the distribution of annual aggregate claims, calculates the premium loading factor necessary such that the probability that annual aggregate claims exceed premium income is 0.05.
[Hint: $\mathbb{P}(S > (1+\theta)E[S]) = 0.05$]

(iii) Market conditions dictate that the insurer can only charge a premium which includes a loading of 25%. Calculate the amount of capital that the insurer must allocate to this line of business in order to ensure that the probability that annual aggregate claims exceed premium income and capital is 0.05. (Use a normal approximation).
[Hint: $\mathbb{P}(S > (1+\theta)E[S] + \text{Capital}) = 0.05$]

(20 marks)

(b) Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has a probability density function

$$f(x) = \begin{cases} \dfrac{x(4-x)}{9} & \text{for } \quad 0 < x < 3, \\ 0 & \text{otherwise,} \end{cases}$$

where x is measured in millions. Calculate the total amount, in millions of ringgits, the insurer would expect to pay under this policy.

(5 marks)

Q3 (a) The number of claims arising from a hurricane in a particular region has a Poisson distribution with mean λ. The amount of claim distribution has mean 0.5 and variance 1.

(i) Determine the mean and variance of the total amount of claims arising from a hurricane.

(ii) The number of hurricanes in this region in one year has a Poisson distribution with mean μ. Determine the mean and variance of the total amount claimed from all the hurricanes in this region in one year.

(14 marks)

(b) Claims occur in a Poisson process rate 20. Individual claims are independent random variables with density

$$f(x) = \frac{3}{(1+x)^4}, \quad x > 0,$$

which is independent of the arrivals process.

(i) Calculate the mean and variance of the total amount claimed by time $t = 2$.

(ii) Using a normal approximation, derive approximately the probability of ruin at $t = 2$ if the premium loading factor is 30% and the initial surplus is $u = 10$.

(6 marks)

Q4 (a) Let $U(t;u)$ be a compound Poisson surplus function (amount of claim) with Gamma distribution $X \sim \mathcal{G}(3, 0.5)$. Compute the adjustment coefficient and its approximate value using equation

$$r^* \simeq \frac{2\theta\mu_X}{\sigma_X^2 + (1+\theta)^2\mu_X^2},$$

for $\theta = 0.1$ and 0.2. Calculate the upper bounds for the probability of ultimate ruin for $u = 5$ and $u = 10$.

(15 marks)

(b) The claims process is a compound Poisson process with rate λ, and individual claim amounts have a Gamma distribution with mean $2\alpha^{-1}$ and variance $2\alpha^{-2}$. A premium loading factor of 50% is used .

(i) Determine the adjustment coefficient.

(ii) Derive an upper bound on the probability of ruin if the initial surplus is u.

(iii) State how this upper bound depends on α, and explain this dependence using general reasoning.

(15 marks)

- END OF QUESTIONS -

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Probability Distributions

Discrete Distributions	p.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Binomial	$\binom{n}{x}p^x q^{n-x}$, $x = 0, 1, \ldots n$	$0 < p < 1$ $q = 1 - p$	$(pe^s + q)^n$	np	npq
Bernoulli	Special case $n = 1$				
Negative Binomial	$\binom{r+x-1}{x}p^r q^x$, $x = 0, 1, 2, \ldots$	$0 < p < 1$ $q = 1 - p$ $r > 0$	$\left(\frac{p}{1 - qe^s}\right)^r$, $qe^s < 1$	$\frac{rq}{p}$	$\frac{rq}{p^2}$
Geometric	Special case $r = 1$				
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$	$\lambda > 0$	$e^{\lambda(e^s - 1)}$	λ	λ
Uniform	$\frac{1}{n}$, $x = 1, \ldots, n$	n, a positive integer	$\frac{e^s(1 - e^{sn})}{n(1 - e^s)}$, $s \neq 0$ 1, $s = 0$	$\frac{n + 1}{2}$	$\frac{n^2 - 1}{12}$

Continuous Distributions	p.d.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Uniform	$\frac{1}{b - a}$, $a < x < b$	—	$\frac{e^{bs} - e^{as}}{(b - a)s}$, $s \neq 0$ 1, $s = 0$	$\frac{b + a}{2}$	$\frac{(b - a)^2}{12}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}}\exp[-(x - \mu)^2/2\sigma^2]$, $-\infty < x < \infty$	$\sigma > 0$	$\exp(\mu s + \sigma^2 s^2/2)$	μ	σ^2
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$, $x > 0$	$\alpha > 0$, $\beta > 0$	$\left(\frac{\beta}{\beta - s}\right)^\alpha$, $s < \beta$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Exponential	Special case $\alpha = 1$				
Chi-square	Special case $\alpha = \frac{k}{2}$, $\beta = \frac{1}{2}$	k, a positive integer			
Inverse Gaussian	$\frac{\alpha}{\sqrt{2\pi\beta}}x^{-3/2}\exp\left[-\frac{(\beta x - \alpha)^2}{2\beta x}\right]$, $x > 0$	$\alpha > 0$, $\beta > 0$	$\exp\left[\alpha\left(1 - \sqrt{1 - \frac{2s}{\beta}}\right)\right]$, $s < \frac{\beta}{2}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Pareto	$\alpha x_0^\alpha / x^{\alpha+1}$, $x > x_0$	$x_0 > 0$, $\alpha > 0$		$\frac{\alpha x_0}{\alpha - 1}$ $\alpha > 1$	$\frac{\alpha x_0^2}{(\alpha - 2)(\alpha - 1)^2}$ $\alpha > 2$
Lognormal	$\frac{1}{x\sigma\sqrt{2\pi}}\exp[-(\log x - m)^2/2\sigma^2]$, $x > 0$	$-\infty < m < \infty$ $\sigma > 0$		$e^{m+\sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2m+\sigma^2}$

Utility Theory

- An insurer with utility $u(\ )$ and wealth w needs π or more to cover X if $E[u(w - \pi - X)] = u(w)$.
- Expected value: $E[w] = pw_1 + qw_2$
- Expected utility function: $E[u(w)] = pu(w_1) + qu(w_2)$

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Individual Risk Model

- Aggregate claim is $S = X_1 + X_2 + \cdots + X_n$ where n is number of risk unit insured and X_i is the distribution of amount of claims
- X, the claim random variable. Its p.f. is

$$f_X(x) = Pr(X = x) = \begin{cases} 1-q, & x = 0 \\ q, & x = b \\ 0, & \text{elsewhere}, \end{cases}$$

where q, the probability of a claim during the year and b, insurer pay amount if the insured dies with in a year of policy issue and nothing if insured survives the year.

- The distribution function is

$$F_X(x) = Pr(X \leq x) = \begin{cases} 0, & x < 0 \\ 1-q, & 0 < x < b \\ 1, & x \geq b \end{cases}$$

- Mean, $E[X] = bq$ and Variance, $Var(X) = b^2 q(1-q)$.
- Conditional Expectations and Variance:

$$E[X] = E[E[X \mid I]] = \mu q,$$

$$Var[X] = Var(E[X \mid I]) + E[Var(X \mid I)] = \mu^2 q(1-q) + \sigma^2 q.$$

- Distribution function of sum of two independent random variables is,

$$F_S(s) = \sum_{all\ y \leq s} F_X(s-y) f_Y(y),$$

and the probability function is

$$f_S(s) = \sum_{all\ y \leq s} f_X(s-y) f_Y(y).$$

- MGF is $M_S(t) = E[e^{tS}]$.

Normal Approximation

- The approximate distribution of S is,

$$Pr(S \leq s) = Pr\left(\frac{S - E(S)}{\sqrt{Var(S)}} \leq \frac{s - E(S)}{\sqrt{Var(S)}}\right)$$

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Collective Risk Model

$$S = \sum_{j=1}^{N} X_j$$

$N, X_1, X_2, \ldots$ are independent random variables
Each X_j has d.f. $P(x)$,
m.g.f. $M_X(t)$, and
$p_k = E[X^k] \quad k = 1, 2, \ldots$

$$P^{*0}(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$P^{*n}(x) = \begin{cases} \sum_{j=0}^{x} p(x-j)P^{*(n-1)}(j), \text{ or} \\ \int_0^x p(x-y)P^{*(n-1)}(y)dy \end{cases}$$

Definitions	Distribution Function, $F_S(x)$	Restrictions on Parameters	Moment Generating Function, $M_S(t)$	Mean	Variance
General	$\sum_{n=0}^{\infty} \Pr(N=n)P^{*n}(x)$		$M_N[\log M_X(t)]$	$p_1 E[N]$	$E[N](p_2 - p_1^2) + p_1^2 \operatorname{Var}(N)$
Compound Poisson	$\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} P^{*n}(x)$	$\lambda > 0$	$e^{\lambda[M_X(t)-1]}$	λp_1	λp_2
Compound Negative Binomial	$\sum_{n=0}^{\infty} \binom{r+n-1}{n} p^r q^n P^{*n}(x)$	$0 < p < 1$, $q = 1 - p$, $r > 0$	$\left[\frac{p}{1 - qM_X(t)}\right]^r$, $qM_X(t) < 1$	$\frac{rqp_1}{p}$	$\frac{rqp_2}{p} + \frac{rq^2p_1^2}{p^2}$
Compound Poisson Inverse Gaussian	no known closed form	$\alpha > 0$, $\beta > 0$	$\exp\left\{\alpha\left[1 - \left\{1 - \frac{2[M_X(t)-1]}{\beta}\right\}^{1/2}\right]\right\}$	$\frac{\alpha}{\beta} p_1$	$\frac{\alpha}{\beta}\left(p_2 + \frac{p_1^2}{\beta}\right)$

Ruin Theory

- Surplus process is $U(t) = u + ct - S(t), t \geq 0$ where $U(t)$, the insurer's random capital at time t; $u = U(0)$, the initial surplus; c, the constant premium income per unit of time; $S(t) = X_1 + X_2 + \cdots + X_{N(t)}$.
- Ruin probability, $\psi(u) = Pr(T < \infty)$ where

$$T = \begin{cases} min\{t \mid t \geq 0 \& U(t) < 0\}; \\ \infty, \ if \ U(t) \geq 0 \ \forall t. \end{cases}$$

- Adjustment coefficient (continuos case): $1 + (1+\theta)\mu R = M_X(R)$
- Loading factor, $c = (1+\theta)\lambda\mu$.

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