UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(TAKE HOME)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	PARTIAL DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 30303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

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THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Show that

$$u(x,y)=\exp\left(-\frac{x}{a}\right)f(bx-ay),$$

where f is an arbitrary function, satisfies the partial differential equation

$$a\frac{\partial u}{\partial x}+b\frac{\partial u}{\partial y}+u=0.$$

By using transformation

$$p=\ln x \quad \text{and} \quad q=\ln y,$$

convert the below equation

$$x\frac{\partial z}{\partial x}+3y\frac{\partial z}{\partial y}-2z=0,$$

into above partial differential equation, hence find the solution.

(9 marks)

(b) Let p be a real number. Consider the partial differential equation

$$xu_x+yu_y=pu, \quad -\infty<x<\infty, \quad -\infty<y<\infty.$$

(i) Find the characteristic curves for the equation.

(4 marks)

(ii) Let $p=4$. Obtain an explicit solution that satisfies $u=1$ on the circle $x^2+y^2=1$.

(3 marks)

(iii) Let $p=2$. Determine two solutions that satisfy $u(x,0)=x^2$, for every $x>0$.

(5 marks)

Q2 (a) Consider the equation

$$xu_{xx}-yu_{yy}+\frac{1}{2}(u_x-u_y)=0.$$

(i) Decide the domain where the equation is elliptic, and the domain where it is hyperbolic.

(2 marks)

(ii) For each of the above to domains, determine the corresponding canonical transformation.

(12 marks)

(b) Given the following partial differential equation

$$2u_{xx} - 3u_{yy} - 4u_x - u_y + 2u = 0.$$

Obtain the values of a and b such that the transformation

$$u(x, y) = \phi(x, y)e^{ax+by}$$

reduces the given partial differential equation into a new partial differential equation in ϕ and this equation does not contain first order derivative terms. Rescale the resulting equation so that all the coefficients are either $+1$ or -1.

(6 marks)

Q3 (a) A thin semicircular plate of radius a has its bounding parameter kept at zero temperature and its curved boundary at a constant temperature T_0. The steady state temperature $T(r,\theta)$ at a point having polar coordinates (r,θ) referred to the centre of the circle as origin, is given by the equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0.$$

Assuming a separated solution of the form $T(r,\theta) = R(r)H(\theta)$, prove that

$$T(r,\theta) = \frac{4T_0}{\pi}\sum_{n=1}^{\infty}\frac{(r/a)^{2n-1}}{(2n-1)}\sin(2n-1)\theta.$$

(10 marks)

(b) Solve for the steady-state temperature distribution in a thin, flat plate covering the rectangle $0 < x < 4$, $0 < y < 1$ if the temperature on the horizontal sides is zero while the temperature on the left side is $f(y) = \sin(\pi y)$ and on the right side, $g(y) = y(1-y)$.

(10 marks)

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Q4 A metal bar of length $l=1$ meter and thermal diffusivity $\gamma=2$ is taken out of a $100^{\circ}C$ oven. Next, the metal bar is fully insulated where its left end maintained at temperature zero while the right end is insulated.

(a) Write an initial-boundary value problem that describes the temperature $u(t,x)$ of the bar at all subsequent times.

(2 marks)

(b) Determine the series formula for the temperature distribution $u(t,x)$ at time $t>0$.

(9 marks)

(c) What is the equilibrium temperature distribution in the bar, i.e. for $t>>0$? How fast does the solution go to equilibrium?

(5 marks)

(d) Just before the temperature distribution reaches equilibrium, what does it look like? Sketch a picture and discuss.

(8 marks)

Q5 Consider the problem of a finite string governed by the one-dimensional wave equation

$$c^2\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 y}{\partial t^2},\quad 0\le x\le 1,\quad t>0,$$

boundary conditions : $y(0,t)=0$ and $y(1,t)=0,\ t>0,$

initial conditions : $y(x,0)=f(x)$ and $\dfrac{\partial y}{\partial t}(x,0)=0,\ 0\le x\le 1,$

where

$$f(x)=\begin{cases} x, & 0\le x\le \dfrac{1}{2}, \\ 1-x, & \dfrac{1}{2}\le x\le 1. \end{cases}$$

(a) Given that the series solution $y(x,t)=\sum_{n=1}^{\infty} b_n \sin(n\pi x)\cos(\pi ct)$ constitutes part of the above solution. Show that the complete solution of the wave equation is given by

$$y(x,t)=\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)\cos(n\pi ct).$$

(3 marks)

(b) Rewrite the solution as a progressive wave and plot the solution in terms of the displacement against distance over the physical interval (0, 1).

(4 marks)

(c) Consider next an infinite string problem, with the same initial conditions as in **Q5(a)** valid on the interval $(-\infty,\infty)$. Obtain the d'Alembert form of solution for this problem and display the solutions via two plots, namely y versus x and a plot of $y(x,t)$ in the $x-t$ plane.

(8 marks)

- END OF QUESTIONS -