Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	OPTIMAL CONTROL
COURSE CODE	:	BWA 31303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS
		OPEN BOOK EXAMINATION

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 (a) Find the minimum cost from node *a* to the desired destination at node *j* in **Figure Q1(a)** and show the optimal path you found.

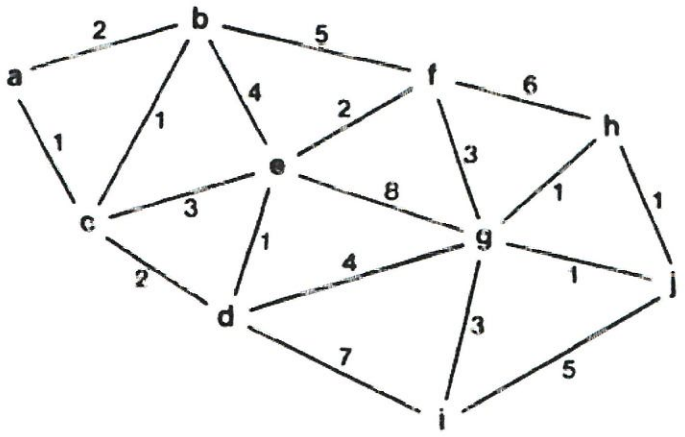

Figure Q1(a): Routing Network

(17 marks)

(b) Construct the Hamiltonian-Jacobi-Bellman equation for a nonlinear optimal control system given as follows

Dynamical system

$$\dot{x} = f(x,u,t),\ x(0) = x_0$$

Performance Index

$$\text{Minimize } J(x(t_0),t_0) - \phi(x(t_f),t_f) + \int_{t_0}^{t_f} L(x,u,t)\ dt$$

(8 marks)

Q2 (a) Calculate the Kalman feedback gain sequences and the Riccati equation solution sequences for the linear control system below

Scalar system

$$x(k+1) = 2x(k) + u(k),\ x(0) = 3$$

Performance index

$$\text{Minimize } J(0) = 5(x(5))^2 + \frac{1}{2}\sum_{k=0}^{4}((x(k))^2 + (u(k))^2)$$

(11 marks)

(b) Recommend the sequences for state, control and cost to the scalar system given in **Q2(a)**.

(14 marks)

Q3 (a) Identify the fundamental results from the Pontryagin minimum principle for a general control system given as follows.

Dynamical system

$$\dot{x} = f(x,u,t),\ x(0) = x_0$$

Performance Index

$$\text{Minimize } J(x(t_0),t_0) - \phi(x(t_f),t_f) + \int_{t_0}^{t_f} L(x,u,t)\ dt$$

(10 marks)

(b) Deduce the optimal control law when the Hamiltonian function is given by

$$H(x,u,\lambda,t) = \frac{u^2}{2} + \lambda(-ax + bu)$$

for some constants a and b, the initial state $x(0) = 0$ and the fixed final state $x(T) = 10$ on the time interval $(0,T)$.

(15 marks)

Q4 (a) A particle of unit mass moves along the x-axis subject to a force $u(t)$. It is required to determine the control which transfers the particle from rest at the origin to rest at $x = 1$ in unit time, so as to minimize the effort involved, measured by

$$\int_0^1 u^2 dt.$$

The equation of motion is $\ddot{x} = u$. Validate the optimal control law for this problem

(13 marks)

(b) Prepare the graph for the trajectories of state, costate, control and cost.

(12 marks)

END OF QUESTIONS