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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	NUMERICAL METHODS II
COURSE CODE	:	BWA 32403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 Evaluate the following double integral using the trapezoid rule with subinterval $n = 4$.

$$\int_{-2}^{2}\int_{0}^{4}(x^2 - 3y^2 + xy^3)\,dy\,dx$$

(15 marks)

Q2 Let $y_1(x)$ is a solution of initial value problem (IVP)

$$y'' = p(x)y' + q(x)y + r(x) \quad , \qquad a \le x \le b,\ y(a) = \alpha\ ,\ y'(a) = 0\ , \qquad (1)$$

and $y_2(x)$ is a solutions of IVP

$$y'' = p(x)y' + q(x)y \quad , \qquad a \le x \le b,\ y(a) = 0\ ,\ y'(a) = 1. \qquad (2)$$

Given the two point boundary value problem (BVP) second order differential linear equation in the form

$$y'' = p(x)y' + q(x)y + r(x) \qquad (3)$$

subject to boundary conditions $a \le x \le b,\quad y(a) = \alpha\ ,\ y(b) = \beta$.

Show that the solution of the original two-point BVP (3) is given by

$$y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)}y_2(x),\ y_2(b) \ne 0.$$

(10 marks)

Q3 (a) Write the Taylor series for function $f(x,t)$ where x is the spatial and t is the time. Hence, derive the first derivative two point forward difference respect to t and second derivative three point central difference formulas for f respect to x.

(10 marks)

(b) Let parabolic equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2},$$

subject to the initial condition $T(x,0) = x^2$ for $0 < x < 1$, and the boundary conditions $T(0,t) = 0$ and $T(1,t) = 1$ for $t > 0$. Find the finite difference equation of the problem using implicit method taking $\Delta x = 0.2$, $t = 0.1$ and $\Delta t = 0.02$.

(15 marks)

Q4 Consider

$$\frac{d^2T}{dx^2} = k(T_a - T), \tag{4}$$

subject to boundary conditions $T(0) = 1$ and $T(1) = e^2$. Solve equation (4) from $x = 0$ to $x = 1$ with $\Delta x = 0.25$, $k = T_a - 1$ based on the following instruction (Use up to 3 decimal places)

(a) Write equation (4) into two initial value problems.

(4 marks)

(b) Solve the set of ordinary differential equations using 4th order Runge-Kutta method with initial conditions $T(0) = 1$ and $z(0) = e$.

(18 marks)

(c) Solve the set of ordinary differential equations using 4th order Runge-Kutta method with $z(0) = 3e$.

(6 marks)

(d) Solve the obtained system using shooting method.

(6 marks)

(e) Solve equation (4) using finite difference method

(14 marks)

(f) Given that the analytical solution of the problem is $T(x) = e^{1+x} - e^{1-x} + 1$, compute the percent error in your numerical calculations.

(2 marks)

END OF QUESTIONS

FINAL EXAMINATION

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Fourth-order Runge-Kutta Method for $\frac{dy}{dx} = f(x, y)$ is given by

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$k_1 = f(x_i\,,\; y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h\,, y_i + \frac{1}{2}k_1 h)$$

$$k_3 = f(x_i + \frac{1}{2}h\,, y_i + \frac{1}{2}k_2 h)$$

$$k_4 = f(x_i + h\,, y_i + k_3 h)$$