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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ALTERNATIVE ASSESSMENT)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	NUMERICAL METHODS I
COURSE CODE	:	BWA 21303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	2 WEEKS
INSTRUCTION	:	ANSWER ALL QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

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Q1 An $n \times n$ tridiagonal system has the following coefficient matrix

$$\begin{bmatrix} a_1 & c_1 & 0 & \cdots & 0 \\ b_2 & a_2 & c_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & c_{n-1} \\ 0 & \cdots & 0 & b_n & a_n \end{bmatrix}.$$

(a) When solving a tridiagonal system with the LU factorization, what is the structure of the resulting L and U factors? Explain why this happens. For this special structure, what modification can be done on the algorithm of the LU factorization and analyze the gain of the modification made.

(10 marks)

(b) Can a tridiagonal system be ill-conditioned? Give an example and demonstrate how the conditioning of the matrix involved affects the accuracy of the solution obtained.

(10 marks)

Q2 (a) By solving an appropriate initial value problem, make a table of the function

$$f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$

on the interval $0 \le x \le 2$. Determine how accurately $f(x)$ is approximated on this interval by the function

$$g(x) = 1 - (ay + by^2 + cy^3)\frac{2}{\sqrt{\pi}}e^{-x^2}$$

where
$a = 0.3084284, b = -0.0849713, c = 0.6627698, y = (1 + 0.47047x)^{-1}$.

(10 marks)

(b) Give an example of a stiff ordinary differential equation. Explain why certain methods fails to give a good approximation to the problem and suggest a way to handle this difficulty.

(20 marks)

Q3 Consider the integral

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, dx.$$

Because the singularities at the endpoints of the interval $[-1,1]$, closed rules cannot be used. Suggest a technique to handle this situation and demonstrate how it works by approximating the given integral. Analyze the error.

(16 marks)

Q4 Consider the $(m-1)\times(m-1)$ tridiagonal matrix

$$A=\begin{bmatrix}1+2\alpha & -\alpha & 0 & \cdots & 0\\ -\alpha & 1+2\alpha & -\alpha & \ddots & \vdots\\ 0 & \ddots & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & \ddots & -\alpha\\ 0 & \cdots & 0 & -\alpha & 1+2\alpha\end{bmatrix}$$

Approximate $\rho(A^{-1})$ with $m=11$ for each of the following values

(a) $\alpha=\frac{1}{4}$.

(b) $\alpha=\frac{1}{2}$.

(c) $\alpha=\frac{3}{4}$.

Explain your approach. Are there ways to improve the convergence?

Verify that the eigenvalues of the matrix A are

$$\lambda_i=1+4\alpha\left(\sin\frac{\pi i}{2m}\right)^2,\ \text{for } i=1,\cdots,m-1.$$

Compare your approximation to the actual value of $\rho(A^{-1})$. Explain your findings.

(20 marks)

Q5 The Shrine of the Book is a wing of the Israel Museum located in Jerusalem, Israel (see **Figure Q5**). The structure houses the Dead Sea Scrolls and has a pagoda-like dome which is reminiscent of the shape of the ancient jars where the scrolls were found in 1947.

Figure Q5: The Shrine of the Book

Use a single polynomial interpolant to approximate the upper outline of The Shrine. Do this by selecting a number of points on The Shrine and record them as data points for the interpolant. Demonstrate the problematic situation which will arise when using the single polynomial interpolant. Recommend a better interpolant and show the improvement achieved.

(14 marks)

– **END OF QUESTIONS** –