Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS I
COURSE CODE	:	BDU 10903
PROGRAMME	:	BDC/BDM
EXAMINATION DATE	:	JANUARY/FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B.**

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

PART A

Q1 (a) Integrate the given integrals over three dimension Cartesian coordinate

$$\int_{-1}^{2}\int_{1}^{z^2}\int_{0}^{y+z} 3yz^2 \, dx\,dy\,dz.$$

(10 marks)

(b) Calculate the volume of the solid that lies inside cone $z = 1 - \sqrt{x^2 + y^2}$ and above the plane $z = -1$ by using cylindrical coordinates.

(10 marks)

Q2 Given a helix, $\underline{r}(t) = 4(\cos t)\,\underline{i} + 4(\sin t)\,\underline{j} + t\,\underline{k}$, $0 \le t \le 4\pi$.

(a) Sketch the helix.

(6 marks)

(b) Find the unit tangent vector at t.

(5 marks)

(c) Determine the curvature for $\underline{r}(t)$.

(5 marks)

(d) Find the arc length for $\underline{r}(t)$.

(4 marks)

PART B

Q3 (a) Find the Maclaurin polynomials p_0, p_1, p_2, p_3 and p_n for $f(x) = \cos x$.

(7 marks)

(b) Investigate and determine the radius of convergence of $\sum_{n=0}^{\infty} 3(x-2)^n$.

(6 marks)

(c) Find a power series for $f(x) = \ln x$, centered at 1.

(7 marks)

Q4 (a) Find the limits

(i) $\lim_{x \to e^2} \dfrac{(\ln x)^3 - 8}{\ln x - 2}$

(4 marks)

(ii) $\lim_{x \to 0^+} \dfrac{\sin x}{5\sqrt{x}}$

(3 marks)

(iii) $\lim_{x \to \infty} \dfrac{\left(1 + 5x^{1/3} + 2x^{5/3}\right)^2}{x^5}$

(4 marks)

(b) Evaluate the value of constants A and B, so that the following function $f(x)$ will be continuous for all x.

$$f(x) = \begin{cases} \dfrac{x^2 - Ax - 6}{x-2}, & x > 2 \\ x^2 + B, & x \leq 2. \end{cases}$$

(5 marks)

(c) Express $\dfrac{(2+i)^2}{2-3i}$ in the form of $a + ib$.

(4 marks)

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Q5 (a) If $y = x + \cos(xy)$, find $\frac{dy}{dx}$.

(4 marks)

(b) Construct $\frac{dy}{dx}$ from the following $x = e^{-t}\cos 2t$ and $y = e^{2t}\sin 2t$.

(6 marks)

(c) Given

$$z = e^{xy}, \quad x = u + v, \quad y = \frac{u}{v}.$$

Differentiate z partially and identify $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

(10 marks)

Q6 (a) The radius of a circle is increasing at the rate of 5 cm per second. Find

(i) the rate of change of the circle area when its radius is 12 cm.
[Hint: Area, $A = \pi r^2$]

(4 marks)

(ii) the radius of the circle when its area is increasing at a rate of 50π cm^{2}s^{-1}.

(2 marks)

(b) A particle P is moving along the x-axis, such that its displacement x at time t is $x(t) = t^2 - 4t$, where t is measured in seconds and $x(t)$ is measured in meters. Find the acceleration of the particle.

(2 marks)

(c) Evaluate $\int \frac{x-3}{3x^2 + 2x - 5}\,dx$ using partial fractions.

(12 marks)

- END OF QUESTIONS -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \text{sech}^2 x \, dx = \tanh x + C$$

$$\int \text{csch}^2 x \, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \; |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \; |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \; |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \; |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \; |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \; 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \; x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

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TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$ $\tan x = \frac{2t}{1-t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$ $dx = \frac{2dt}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$ $\tan 2x = \frac{2t}{1-t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$ $dx = \frac{dt}{1+t^2}$

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IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$
$$\cos(x + y) = \cos x\cos y \mp \sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

CURVATURE, ARC LENGTH AND TANGENT VECTORS

$$\kappa = \frac{\|dT/dt\|}{\|d\underline{r}/dt\|} \qquad s(t) = \int_a^b \|\underline{r}'(t)\|dt \qquad \underline{T}(t) = \frac{\underline{r}'(t)}{\|\underline{r}'(t)\|}, \quad \underline{r}'(t) \neq 0$$

Cylindrical coordinates:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \quad x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$$
$$V = \iiint_G dV = \iiint_G dz\, r\, dr\, d\theta$$