UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	(a) ANSWER **ALL** QUESTIONS IN **PART A** (b) ANSWER **TWO (2)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 (a) State a suitable numerical method that can be used to determine
(i) The largest eigenvalue
(ii) The smallest eigenvalue

(2 marks)

(b) Given

$$M = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$$

(i) Evaluate the largest eigenvalue and its corresponding eigenvector using an appropriate numerical method. Use $x_0 = (A \quad 0 \quad B)^T$ and iterate until $|\lambda_{k+1} - \lambda_k| \le 0.01$. Note that A is the first digit of your matrix number and B is the last digit of your matrix number.

(10 marks)

(ii) Validate your answer in **Q1(b)(i)** with reference to the solution given by the characteristic equation in terms of absolute error.

(8 marks)

Q2 The equation of performance for a machine system under an inspection is expressed as:

$$y'' + 3y' = y + x^2$$

Given the boundary condition for the system as $y(0) = 2$ and $y(2) = 5$. As an engineer, you are responsible to check the system's performance at the following point: $x = 0.5, 1$ and 1.5.

(a) By using central finite difference approximation, prove that the differential equation can be written as $y_{i-1} - 9y_i + 7y_{i+1} = x_i^2$.

(7 marks)

(b) Considering the boundary condition, deconstruct the differential equation into a matrix form.

(9 marks)

(c) Solve for the unknown at $x = 0.5, 1$ and 1.5.

(4 marks)

Q3 A 2 meter long metal plate with thermal diffusivity, $\kappa = 5m^2/s$ is insulated on its side such that the heat transfer only occurs in axial direction. The left end of the plate is maintained at A°C, while the right end of the plate is convectively cooled by a fluid at B°C for $t > 0$. The temperatures on both ends are maintained for 2 seconds. The distribution of the initial temperature is given by $T(x,0) = 2x^2$.

Note that A is the first three digits of your matrix number and B is the last two digits of your matrix number.

The unsteady state heat conduction equation is given by

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

(a) Draw the finite difference grid to predict the temperature of all points up to 2 seconds, if given $\Delta x = 0.5$ and $\Delta t = 1$. Label all unknown temperatures on the grid.

(5 marks)

(b) Transform the unsteady state heat conduction equation into a system of linear equations using implicit Crank-Nicolson method for every 1 second.

(12 marks)

(c) Determine the unknown temperatures at $t = 1$ second.

(3 marks)

PART B: ANSWER TWO (2) QUESTIONS

Q4 (a) Two cars A and B move on the road. The position of car A at any instant of time is given by $s(t) = 0.95t^3 - 5.9t^2 + C$ and the position of car B at any instant of time is given by $s(t) = 6 - 10.9t + D$.

Note that C is the last two digits of your matrix number and D is the last three digits of your matrix number

Estimate the time when both cars arrive at the same position using

(i) Newton Raphson method, with initial $t_0 = 3.5$. Perform your calculation until third iteration.

(6 marks)

(ii) Secant method, with the interval [2.5,3.5]. Perform your calculation until fourth iteration.

(4 marks)

(b) Given the following system of linear equations:

System 1: $$\begin{pmatrix} A & 2 & 3 & 0 \\ 9 & B & 0 & 1 \\ 1 & C & 0 & 9 \\ 0 & D & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \\ 8 \end{pmatrix}$$

System 2: $$\begin{pmatrix} A & B & 0 & 0 \\ 3 & -9 & 7 & 0 \\ 0 & C & -2 & 8 \\ 0 & 0 & D & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 5 \\ 9 \end{pmatrix}$$

System 3: $$\begin{pmatrix} A & B & 1 \\ C & D & -17 \\ 0 & 8 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \\ 0 \end{pmatrix}$$

Note that:
A – First digit of your matrix number
B – Second digit of your matrix number
C – Last digit of your matrix number
D – Second last digit of your matrix number

(i) Differentiate the system of linear equations into two different categories.

(2 marks)

(ii) From System 1 to System 3, select the system of linear equations that can be solved by Thomas Algorithm and solve it subsequently.

(8 marks)

Q5 (a) **Table Q5(a)** is generated from the function of $f(x) = e^x - 2x^2 + A$, where the value of A is the last two digits of your matrix number.

Table Q5(a): Data generated from function $f(x) = e^x - 2x^2 + A$

x	0	0.5	1	1.5	2
$f(x)$	1+A	1.1487+A	0.7183+A	-0.0183+A	-0.6109+A

Comparing 2-point forward difference and 2-point backward difference, investigate which method gives the highest accuracy in approximating $f'(1.5)$

(10 marks)

(b) Given

$$M = \begin{pmatrix} A & B & 10 \\ 4 & C & 2 \\ 8 & 5 & 3 \end{pmatrix}$$

Note that A represents the first digit of your matrix number, B represents the second digit of your matrix number and C represents the last digit of your matrix number. Evaluate the largest eigenvalue and its corresponding eigenvector using Power method. Use $x_0 = (1 \quad 0 \quad 1)^T$. Perform the calculation until seventh iteration.

(10 marks)

Q6 (a) Solve the initial value problem $y' = \frac{x}{y}, y(0) = 1$ at $x = 0(0.4)2$ using fourth-order Runge Kutta method. If the exact solution is $y = \sqrt{x^2 + 1} + A$, find the absolute errors. Note that the value of A is the last two digits of your matrix number.

(10 marks)

(b) For a function f, the divided-differences are given as in **Table Q6(b)**.

Table Q6(b) Divided differences for function f

$x_0 = 0.0$	$f(x_0) = ?$	$f_0^{[1]} = ?$	$f_0^{[2]} = 7.5 + C$
$x_1 = 0.4$	$f(x_1) = ?$	$f_1^{[1]} = 12 + B$	
$x_2 = 0.7$	$f(x_2) = 7 + A$		

Note that:
A – First digit of your matrix number
B – Second digit of your matrix number
C – Last two digits of your matrix number

(i) Determine the missing entries in Table Q6(b).

(6 marks)

(ii) Determine the value of $f(0.25)$.

(2 marks)

(iii) Your friend, Rangga claims that the above data can be used to estimate $f(0.85)$. Do you agree with him? Justify your answer.

(2 marks)

-END OF QUESTIONS-

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FORMULA

Newton Raphson Method: $x_{i+1} = x_i - \dfrac{y(x_i)}{y'(x_i)}$

Secant Method: $x_{i+1} = \dfrac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$

Thomas Algorithm:

i	1	2	...	n
d_i				
e_i c_i b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Newton's Divided Difference Interpolating Polynomial:

$y(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$

2-Point Forward Difference: $y'(x) = \dfrac{y(x+h) - y(x)}{h}$

2-Point Backward Difference: $y'(x) = \dfrac{y(x) - y(x-h)}{h}$

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FORMULA

Power Method: $\{V\}^{k+1} = \dfrac{[A]\{V\}^{k}}{\lambda_{k+1}}$

Inverse Power Method: $\{V\}^{k+1} = \dfrac{[A]^{-1}\{V\}^{k}}{\lambda_{k+1}}$

Characteristic Equation: det(A-λI)=0

Fourth Order Runge Kutta Method:

$f(x_i, y_i) = y'(x_i)$

$y_{i+1} = y_i + \dfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_i, y_i)$

$k_2 = hf(x_i + \dfrac{h}{2}, y_i + \dfrac{k_1}{2})$

$k_3 = hf(x_i + \dfrac{h}{2}, y_i + \dfrac{k_2}{2})$

$k_4 = hf(x_i + h, y_i + k_3)$

Numerical Differentiation: $y_i' = \dfrac{y_{i+1} - y_{i-1}}{2h}$ $\quad y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Implicit Crank Nicolson Method