



UTHM

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021**

COURSE NAME : ENGINEERING MATHEMATICS IV
COURSE CODE : BDA 34003
PROGRAMME CODE : BDD
EXAMINATION DATE : JANUARY / FEBRUARY 2021
DURATION : 3 HOURS
INSTRUCTION : (a) ANSWER ALL QUESTIONS IN
PART A
(b) ANSWER TWO (2) QUESTIONS
IN PART B

THIS QUESTION PAPER CONSISTS OF EIGHT (8) PAGES

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PART A: ANSWER ALL QUESTIONS

- Q1 (a) State a suitable numerical method that can be used to determine
- The largest eigenvalue
 - The smallest eigenvalue

(2 marks)

- (b) Given

$$M = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$$

- Evaluate the largest eigenvalue and its corresponding eigenvector using an appropriate numerical method. Use $x_0 = (A \ 0 \ B)^T$ and iterate until $|\lambda_{k+1} - \lambda_k| \leq 0.01$. Note that A is the first digit of your matrix number and B is the last digit of your matrix number.
- Validate your answer in Q1(b)(i) with reference to the solution given by the characteristic equation in terms of absolute error.

(10 marks)

(8 marks)

- Q2 The equation of performance for a machine system under an inspection is expressed as:

$$y'' + 3y' = y + x^2$$

Given the boundary condition for the system as $y(0) = 2$ and $y(2) = 5$. As an engineer, you are responsible to check the system's performance at the following point: $x = 0.5, 1$ and 1.5 .

- By using central finite difference approximation, prove that the differential equation can be written as $y_{i-1} - 9y_i + 7y_{i+1} = x_i^2$.
- Considering the boundary condition, deconstruct the differential equation into a matrix form.
- Solve for the unknown at $x = 0.5, 1$ and 1.5 .

(7 marks)

(9 marks)

(4 marks)

- Q3** A 2 meter long metal plate with thermal diffusivity, $\kappa = 5m^2/s$ is insulated on its side such that the heat transfer only occurs in axial direction. The left end of the plate is maintained at $A^\circ C$, while the right end of the plate is convectively cooled by a fluid at $B^\circ C$ for $t > 0$. The temperatures on both ends are maintained for 2 seconds. The distribution of the initial temperature is given by $T(x, 0) = 2x^2$.

Note that A is the first three digits of your matrix number and B is the last two digits of your matrix number

The unsteady state heat conduction equation is given by

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

- (a) Draw the finite difference grid to predict the temperature of all points up to 2 seconds, if given $\Delta x = 0.5$ and $\Delta t = 1$. Label all unknown temperatures on the grid. (5 marks)
- (b) Transform the unsteady state heat conduction equation into a system of linear equations using implicit Crank-Nicolson method for every 1 second. (12 marks)
- (c) Determine the unknown temperatures at $t = 1$ second. (3 marks)

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PART B: ANSWER TWO (2) QUESTIONS

- Q4** (a) Two cars A and B move on the road. The position of car A at any instant of time is given by $s(t) = 0.95t^3 - 5.9t^2 + C$ and the position of car B at any instant of time is given by $s(t) = 6 - 10.9t + D$.

Note that C is the last two digits of your matrix number and D is the last three digits of your matrix number

Estimate the time when both cars arrive at the same position using

- (i) Newton Raphson method, with initial $t_0 = 3.5$. Perform your calculation until third iteration. (6 marks)
- (ii) Secant method, with the interval $[2.5, 3.5]$. Perform your calculation until fourth iteration. (4 marks)

- (b) Given the following system of linear equations:

$$\text{System 1: } \begin{pmatrix} A & 2 & 3 & 0 \\ 9 & B & 0 & 1 \\ 1 & C & 0 & 9 \\ 0 & D & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \\ 8 \end{pmatrix}$$

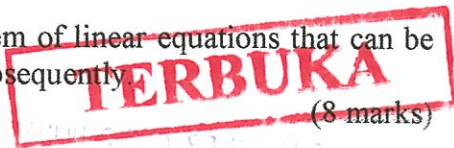
$$\text{System 2: } \begin{pmatrix} A & B & 0 & 0 \\ 3 & -9 & 7 & 0 \\ 0 & C & -2 & 8 \\ 0 & 0 & D & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 5 \\ 9 \end{pmatrix}$$

$$\text{System 3: } \begin{pmatrix} A & B & 1 \\ C & D & -17 \\ 0 & 8 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \\ 0 \end{pmatrix}$$

Note that:

- A – First digit of your matrix number
- B – Second digit of your matrix number
- C – Last digit of your matrix number
- D – Second last digit of your matrix number

- (i) Differentiate the system of linear equations into two different categories. (2 marks)
- (ii) From System 1 to System 3, select the system of linear equations that can be solved by Thomas Algorithm and solve it subsequently. (8 marks)



- Q5** (a) **Table Q5(a)** is generated from the function of $f(x) = e^x - 2x^2 + A$, where the value of A is the last two digits of your matrix number.

Table Q5(a): Data generated from function $f(x) = e^x - 2x^2 + A$

x	0	0.5	1	1.5	2
$f(x)$	$1+A$	$1.1487+A$	$0.7183+A$	$-0.0183+A$	$-0.6109+A$

Comparing 2-point forward difference and 2-point backward difference, investigate which method gives the highest accuracy in approximating $f'(1.5)$

(10 marks)

- (b) Given

$$M = \begin{pmatrix} A & B & 10 \\ 4 & C & 2 \\ 8 & 5 & 3 \end{pmatrix}$$

Note that A represents the first digit of your matrix number, B represents the second digit of your matrix number and C represents the last digit of your matrix number. Evaluate the largest eigenvalue and its corresponding eigenvector using Power method. Use $x_0 = (1 \ 0 \ 1)^T$. Perform the calculation until seventh iteration.

(10 marks)



- Q6 (a)** Solve the initial value problem $y' = \frac{x}{y}, y(0) = 1$ at $x = 0(0.4)2$ using fourth-order Runge Kutta method. If the exact solution is $y = \sqrt{x^2 + 1} + A$, find the absolute errors. Note that the value of A is the last two digits of your matrix number. (10 marks)

- (b) For a function f , the divided-differences are given as in **Table Q6(b)**.

Table Q6(b) Divided differences for function f

$x_0 = 0.0$	$f(x_0) = ?$	$f_0^{[1]} = ?$	$f_0^{[2]} = 7.5 + C$
$x_1 = 0.4$	$f(x_1) = ?$	$f_1^{[1]} = 12 + B$	
$x_2 = 0.7$	$f(x_2) = 7 + A$		

Note that:

A – First digit of your matrix number

B – Second digit of your matrix number

C – Last two digits of your matrix number

- (i) Determine the missing entries in Table Q6(b). (6 marks)
- (ii) Determine the value of $f(0.25)$. (2 marks)
- (iii) Your friend, Rangga claims that the above data can be used to estimate $f(0.85)$. Do you agree with him? Justify your answer. (2 marks)

-END OF QUESTIONS-

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FORMULA

Newton Raphson Method: $x_{i+1} = x_i - \frac{y(x_i)}{y'(x_i)}$

Secant Method: $x_{i+1} = \frac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$

Thomas Algorithm:

<i>i</i>	1	2	...	<i>n</i>
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i \beta_{i-1}$				
$\beta_i = \frac{e_i}{\alpha_i}$				
$y_1 = \frac{b_1}{\alpha_1}$ $y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Newton's Divided Difference Interpolating Polynomial:

$y(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$

2-Point Forward Difference: $y'(x) = \frac{y(x+h) - y(x)}{h}$

2-Point Backward Difference: $y'(x) = \frac{y(x) - y(x-h)}{h}$



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FORMULA

Power Method: $\{V\}^{k+1} = \frac{[A]\{V\}^k}{\lambda_{k+1}}$

Inverse Power Method: $\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$

Characteristic Equation: $\det(A-\lambda I)=0$

Fourth Order Runge Kutta Method:

$f(x_i, y_i) = y'(x_i)$

$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_i, y_i)$

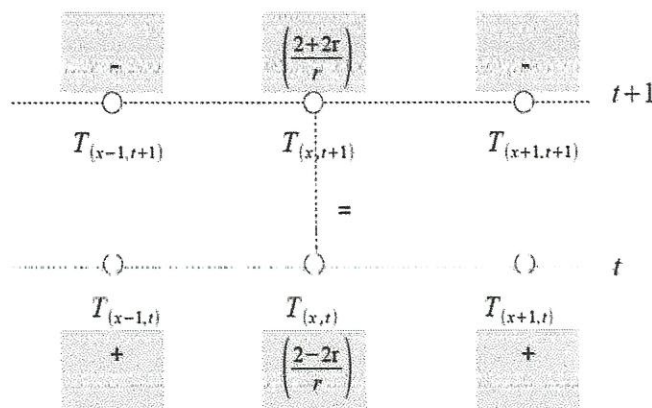
$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$

$k_4 = hf(x_i + h, y_i + k_3)$

Numerical Differentiation: $y_i' = \frac{y_{i+1} - y_{i-1}}{2h}$ $y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Implicit Crank Nicolson Method



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