UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME	:	BDD
EXAMINATION DATE	:	JANUARY/FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTION ONLY

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

Q1 (a) Given that,

$$f(x,y)=\frac{2}{\sqrt{3x^2+9y^2}}$$

(i) Solve the domain and range for the function $f(x,y)$

(2 marks)

(ii) Sketch the contour map of K = 1, $\sqrt{9}$ and 4

(6 marks)

(iii) Sketch 2D and 3D model

(2 marks)

(b) Given the function, $f(x,y) - 2x^2 \quad y^3 \quad 2xy$

(i) Solve all the critical points of function $f(x,y)$

(5 marks)

(ii) Differential each critical point whether as a local maximum, local minimum or saddle point

(5 marks)

Q2 (a) Given $w(x,y,z)=2xyz, x=s^2+t^2, y=\frac{s}{t}, and z=\ln t$, solve $\frac{\delta w}{\delta s}$ and $\frac{\delta w}{\delta t}$

(10 marks)

(b) Use the total differential dz to approximate the change in $z=\sqrt{6-x^2-y^2}$ as (x,y) moves from the point (1,1) to the point (0.99, 1.02). Compare this approximation change with the exact change in z.

(10 marks)

Q3 (a) Distinguish the integral to polar coordinates and calculate the integral.

$$\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}}\frac{1}{\sqrt{1+x^2+y^2}}\,dxdy$$

(5 marks)

(b) Sketch the region R bounded by the graphs of $x=(y-1)^2, y=1, y=3$, and y-axis and use double integral to find the region R

(5 marks)

(c) Distinguish double integrals to calculate the volume of the tetrahedron, $3x+2y+4z=12$, in the first octant.

(5 marks)

(d) Solve the triple integration to calculate the volume of the solid G that is bounded above the hemisphere $z=\sqrt{25-x^2-y^2}$, below by the xy-plane and laterally by the cylindrical $x^2+y^2=9$.

(5 marks)

Q4 (a) Evaluate this double integral without using polar coordinates

$$\int_0^{\ln 2}\int_0^1 xye^{xy^2}\,dydx$$

(5 marks)

(b) Distinguish the surface area of the portion of the paraboloid $z=4-x^2-y^2$ over the xy plane by polar coordinates

(5 marks)

(c) Solve double integrals to calculate the volume of the solid G bounded by the cylinder, $x^2+y^2=4$ and the plane $y+z=4$ and $z=0$. Verify your answer by using triple integrals in cylindrical coordinate method.

(10 marks)

Q5 (a) Find the vector–valued function that represents the curve of intersection of the cylinder $x^2+z^2=5$ and the plane $y+z=1$. Then sketch the curve of intersection.

(4 marks)

(b) A particle's position at time $t=\pi$ is determined by the vector $r(t)=e^{\frac{t}{4}}i+\sin 2tj+t^4k$. Distinguish the particle's motion. Find

(i) the velocity, V

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(2 marks)

(ii) the speed

(2 marks)

(iii) the acceleration, A

(2 marks)

(iv) the direct of motion

(2 marks)

(c) Given vector–valued function, $r(t) = 3\sin ti + 3\cos tj + 3tk$. Distinguish

(i) the unit tangent vector T(t)

(2 marks)

(ii) the principal unit normal vector N(t)

(2 marks)

(iii) the curvature

(2 marks)

(iv) the arc length of the curve in the interval, $0 \le t \le 2$

(2 marks)

Q6 (a) Evaluate surface integral of $F(x, y, z) = xyi + zj + (x + y)k$ over the surface of the tetrahedron $x + y + z = 1$ in the first octant (surface S_1), with the oriented outward unit normal vector.

(5 marks)

(b) Given the force field, $F(x, y, z) = \frac{5}{3} y^3 i + 5xy^2 j + 2k$

(i) Prove that F is conservative

(2 marks)

(ii) By using formula ∇∅ = **F**, test a scalar potential ∅ for F.

(3 marks)

(iii) Hence, compute the amount of work done against the force field F in moving an object from the point (1, 1, 1) to (2, 3, 4).

(2 marks)

(c) Distinguish Strokes theorem to evaluate $\oint_C F.dr$ if $F(x,y,z) = -3yi + 3xj + z^2k$, where C is the boundary of the portion of ellipsoid $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$

(8 marks)

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FORMULAS

Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)

c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)

d. If $D = 0$
   The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS$$

$$= \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)\,dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\,\delta(x,y)\,dA$

b. About y-axis, $M_y = \iint_R x\,\delta(x,y)\,dA$

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Centre of Mass
Non Homogeneous Lamina:

$$(x, y) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

Centroid
Homogeneous Lamina:

$$x = \frac{1}{Area\,of\quad R}\iint_R x\,dA \quad \text{and} \quad y = \frac{1}{Area\,of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $$I_y = \iint_R x^2\delta(x,y)\,dA$$

b. $$I_x = \iint_R y^2\delta(x,y)\,dA$$

c. $$I_o = \iint_R (x^2+y^2)\delta(x,y)\,dA$$

In 3-D: Solid
Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$

If $\delta(x,y,z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

b. About *xz*-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

c. About *xy*-plane, $M_{xy} = \iiint_G z\delta(x,y,z)\,dV$

Centre of Gravity

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t)=\dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t)=\dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector**, $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$

Curvature

$$\kappa=\frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho=\frac{1}{\kappa}$$

Green's Theorem

$$\oint_C Mdx+Ndy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}dS=\iiint_G \nabla\cdot\mathbf{F}dV$$

Stoke's Theorem

$$\oint_C \mathbf{F}\cdot dr=\iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}dS$$

Arc Length

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}, t\in[a,b]$, hence, the **arc length**,

$$s=\int_a^b\|\mathbf{r}'(t)\|\,dt=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$