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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ELECTROMECHANICAL AND CONTROL SYSTEMS
COURSE CODE	:	BDU 20302
PROGRAMME	:	BDC / BDM
EXAMINATION DATE	:	JANUARY/FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

Q1 (a) The roll autopilot of a jet fighter is shown in **Figure Q1(a)**. Determine the closed-loop transfer function $\phi(s)/\phi_d(s)$ if $K_g = 1$.

(5 marks)

(b) Determine the roots of the characteristic equations for controller gain $K = 0.7$, 3 and 6.

(3 marks)

(c) Using the concept of dominant roots, give comments on the effect of gain selection (i.e. $K = 0.7$, 3 and 6) towards the approximate second order system's time response. Suggest the range of gain values that will make the control system unstable.

(9 marks)

(d) Determine a suitable controller gain, K so that the percentage overshoot of the closed-loop system is equal to 16%. Calculate the resulting peak time.

(8 marks)

Q2 (a) List the characteristics of the short period and phugoid stability modes.

(5 marks)

(b) Consider an aircraft model in a wind tunnel setup where the aircraft is constrained at its center of gravity. The aircraft is free to perform a pitching motion about its center of gravity. The governing equation of this simple motion is obtained from Newton's second law and is given as:

$$\Delta\ddot{\alpha} - \left(M_q + M_{\dot{\alpha}}\right)\Delta\dot{\alpha} - M_\alpha \Delta\alpha = M_{\delta e}\Delta\delta_e$$

where $\Delta\alpha$ is the change in the angle of attack (Assumption: the change in the angle of attack and pitch angles are identical), $\Delta\delta e$ is the change in elevator angle. Derivatives, M_q and M_α are the longitudinal derivatives due to pitching velocity and angle of attack. Find the transfer function relating the change in the angle of attack, $\Delta\alpha(s)$ and the change in elevator angle $\Delta\delta e(s)$. Use the Laplace transform theorem in **Table Q2(b)**.

(2 marks)

(c) Determine the solution, $\alpha(t)$ for the governing equation in **Question Q2(b)** if a step input is applied to the elevator using the following data:

$$M_q = -2.05\ s^{-1}$$
$$M_\alpha = -8.80\ s^{-2}$$
$$M_{\dot{\alpha}} = -0.95\ s^{-2}$$
$$M_{\delta e} = -5.5\ s^{-2}$$

Use partial fraction and inverse Laplace theorem to obtain the output response of the system, $\alpha(t)$ with the initial conditions of $\alpha(0) = 0$ and $\frac{d\alpha(0)}{dt} = 0$. Analyze the effect of parameter $\left(M_q + M_{\dot{\alpha}}\right)$ and M_a on the pitching motion.

(18 marks)

Q3 A simplified pitch control system is shown in **Figure Q3** with transfer functions for each component in the control system are given as:

$$K(s) = K_P + \frac{K_I}{s} + K_D s$$

$$G_1(s) = \frac{10}{s+10}$$

$$G_2(s) = \frac{3}{s^2+3s+4}$$

(a) Examine the locus movement of the open loop transfer function in a root locus plot and find the damped frequency, ω_d and gain, K, values at the imaginary axis crossing if such a situation exists.

(10 marks)

(b) Calculate the controller gains for the dynamic system under consideration using the Ziegler-Nichols tuning method with P, PD, and 'No overshoot' PID control.

(8 marks)

(c) Compare the steady-state error performance of the compensated systems (i.e., P, PD, and 'No overshoot' PID control). Describe any problems with your design.

(7 marks)

Q4 (a) Describe the physical characteristics of Dutch Roll stability mode.

(3 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -7.1$ ft/s^2 $\quad$ $Y_r = 2.1$ ft/s

$N_\beta = 2.9$ s^{-2} $\quad$ $N_r = -0.325$ s^{-1}

$Y_{\delta r} = -4.9$ ft/s^2 $\quad$ $N_{\delta r} = 0.615$ s^{-2}

$u_0 = 160$ ft/s

(i) Determine the characteristic equation of the Dutch Roll mode.

(4 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude, and the number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) The roll angle to aileron input transfer function can be modelled according to:

$$\frac{\phi(s)}{\delta_a(s)} = \frac{L_{\delta a}}{s(s - l_p)}$$

Design a roll attitude control system to maintain a wings level attitude for a vehicle having the following characteristics:

$$L_{\delta a} = 2.5/s^2$$
$$L_p = -0.6/s$$

The system performance is to have a damping ratio, $\xi = 0.7022$ and undamped natural frequency, $\omega_n = 5$ rad/s. Consider the sensor used in the control system design to be a perfect device.

(11 marks)

Q5 An attitude control system for a satellite vehicle within the earth's atmosphere is shown in **Figure Q5**. The transfer functions of the system are given as follows:

$$G(s) = \frac{K(s + 0.2)}{(s + 0.9)(s - 0.6)(s - 0.1)}$$

$$G_c(s) = \frac{(s^2 + 4s - 6.25)}{(s + 4)}$$

(a) Draw the detailed root locus plot for the closed-loop system as K varies from 0 to ∞. Provide necessary calculation such as the asymptote angle, centroid, break-in/out, angle of departure/arrival or imaginary axis intersection point to support your answer.

(20 marks)

(b) Suggest a range of gain, K that results in a system with settling time less than 12 s and a damping ratio for the complex roots greater than 0.5.

(5 marks)

-END OF QUESTION-

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Table Q2(b) The Laplace transform theorems

Laplace transforms Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t+\theta)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t+\theta)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$	$t\sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t\cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}[1-e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t-t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s}+\frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s)-f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-s^{n-3}f''(0)-\cdots-f^{n-1}(0)$		

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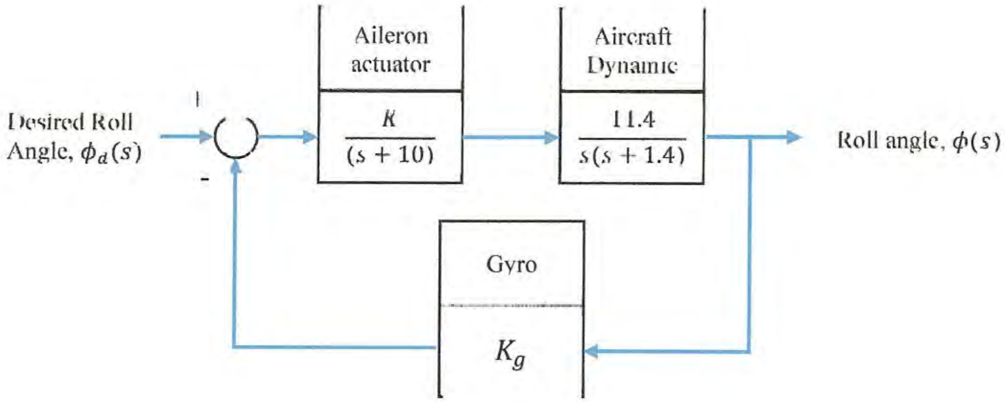

Figure Q1(a) Roll angle control system.

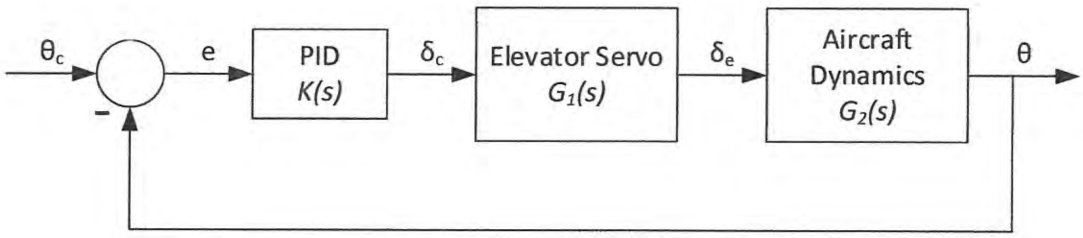

Figure Q3 Simplified block diagram for pitch angle control system.

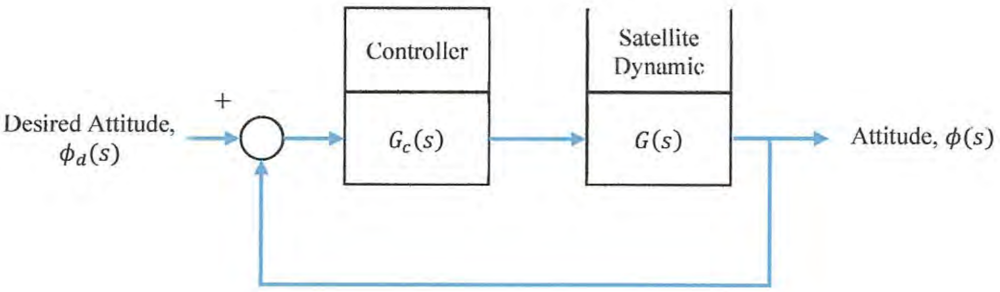

Figure Q5 The block diagram for the satellite control system.

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Key Equations

The relevant equations used in this examination are given as follows

1. The determinant of a 3 × 3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$

4. General first order transfer function:

$$G(s) = \frac{s}{s+a} \quad (4)$$

5. General second order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$

6. The closed loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open-loop system, and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

$$T_r = \frac{2.2}{a} \quad (8)$$

$$T_s = \frac{4}{a} \quad (9)$$

$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

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$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega} \quad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\eta} \quad (13)$$

$$P = \frac{2\pi}{\omega} \quad (14)$$

$$t_{1/2} = \frac{0.693}{|\eta|} \quad (15)$$

$$N_{1/2} = 0.110\frac{|\omega|}{|\eta|} \quad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \quad (17)$$

10. Estimation of the integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix ***M*** and ***N*** are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + A\Delta t + \frac{1}{2!}A^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}A\Delta t + \frac{1}{3!}A^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (20)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum \textit{Real parts of the poles} - \sum \textit{Real parts of the zeros}]}{n-m} \quad (21)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (22)$$

14. The solution to determine real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \quad (23)$$

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15. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \tag{24}$$

16. The angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180^\circ + \sum(angles\ to\ zeros) - \sum(angles\ to\ poles) \tag{25}$$

17. The angle of arrival at a zero:

$$\theta = 180^\circ - \sum(angles\ to\ zeros) + \sum(angles\ to\ poles) \tag{26}$$

18. The steady state error:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \tag{27}$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \tag{28}$$

19. Linear system (SISO) with state feedback:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T)\boldsymbol{x} + \boldsymbol{B}u$$

$$\text{or, } \dot{\boldsymbol{x}} = \boldsymbol{A}_{new}\boldsymbol{x} + \boldsymbol{B}u \tag{29}$$

where $\boldsymbol{A}_{new}$ is the augmented matrix and $u = \boldsymbol{K}^T\boldsymbol{x} + \delta_{ref}$

20. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (30)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$

21. The calculation of controller gains using the Ziegler-Nichols method: (31)

Table 1 The Ziegler-Nichols tuning method.

Control Type	K_p	K_I	K_D
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\,K_p/T_u$	-
PD	$0.8K_u$	-	$K_PT_u/8$
Classic PID	$0.6K_u$	$2\,K_p/T_u$	$K_PT_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5\,K_p/T_u$	$3K_PT_u/20$
Some Overshoot	$0.33K_u$	$2\,K_p/T_u$	$K_PT_u/3$
No Overshoot	$0.2K_u$	$2\,K_p/T_u$	$K_PT_u/3$

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22. The contribution of the wing-body to M_{cg}:

$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac_{wb}})$$
(32)
$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + a_{wb}\alpha_{wb}(h - h_{ac_{wb}})$$

23. The contribution of the wing body tail to M_{cg}.

$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + a\alpha_a\left(h - h_{ac_{wb}} - V_H\frac{a_t}{a}\left[1 - \frac{\partial\varepsilon}{\partial\alpha}\right]\right) + V_H a_t(i_t + \varepsilon_0) \qquad (33)$$

24. The equation for longitudinal static stability.

$$C_{M,0} = C_{M,ac_{wb}} + V_H a_t(i_t + \varepsilon_0)$$
(34)
$$\frac{\partial C_{M,cg}}{\partial\alpha_a} = a\left[h - h_{ac_{wb}} - V_H\frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right]$$

25. The absolute angle of attack, α_a:

$$\alpha_a = \alpha + |\alpha_{L=0}| \qquad (35)$$

where α is the geometric angle of attack.

26. Neutral point:

$$h_n = h_{ac_{wb}} + V_H\frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (36)$$

27. Static margin:

$$SM = h_n - h \qquad (37)$$

28. Elevator angle to trim:

$$\delta_{trim} = \frac{C_{M,0} + (\partial C_{M,cg}/\partial\alpha_a)\alpha_n}{V_H(\partial C_{L,t}/\partial\,\delta_e)} \qquad (38)$$

29. Conversion from the state-space model to transfer function model. (39)

$$G(s) = \boldsymbol{C}\frac{adj(s\boldsymbol{I} - \boldsymbol{A})}{\det(s\boldsymbol{I} - \boldsymbol{A})}\boldsymbol{B}$$