CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	DIFFERENTIAL EQUATIONS
COURSE CODE	:	BDA 24303
PROGRAMME	:	BDD
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTIONS	:	**PART A**: ANSWER **ONLY THREE (3)** FROM **FOUR (4)** QUESTIONS **PART B**: ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF NINE (9) PAGES

CONFIDENTIAL

PART A: Answer only **THREE (3)** from **FOUR (4)** questions

Q1 (a) Show that the following equation is homogeneous and then find the general solution:

$$\frac{dy}{dx} = \frac{3y^2+x^2}{2xy}$$

(6 marks)

(b) Solve the following initial value problems:

(i) $(2xy - 9x^2) + (x^2 + 2)\frac{dy}{dx} = 0\,; \quad y(0) = 3$

(ii) $t\frac{dy}{dt} + 2y = 2 + 3\sin t; \quad y(1) = 2$

(14 marks)

Q2 (a) Using the method of undetermined coefficient, find the general solution to the differential equation:

$$y'' + 9y - 2\cos 3x = 0$$

(9 marks)

(b) Solve the following second order equation:

$$y'' + 2y' - 15y = \frac{2x}{e^{3x}}$$

(11 marks)

Q3 (a) Determine the Laplace Transform of the step function represented by the graph shown in **Figure Q3(a)**

(10 marks)

(b) Obtain the inverse Laplace transform for the following function:

$$\frac{s+1}{(s^2-1)} + \frac{1}{s^2(s^2-1)}$$

(10 marks)

Q4 (a) Determine the inverse Laplace transform of:

$$\frac{20}{(s^2+4)(s^2+2s+2)}$$

(5 marks)

(b) Express the following step function in terms of unit step function and then find the Laplace transform

$$r(t) = \begin{cases} 10\sin 2t, & 0 < t < \pi \\ 0, & \pi < t \end{cases}$$

(6 marks)

(c) Solve the initial value problem for a damped mass spring system below:

$$y'' + 2y' + 2y = r(t)\,; \qquad y(0) = 1, y'(0) = -5$$

$$r(t) = \begin{cases} 10\sin 2t, & 0 < t < \pi \\ 0, & \pi < t \end{cases}$$

(9 marks)

PART B: Answer all questions

Q5 A periodic function is defined as:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

(a) Sketch the graph of $f(x)$ in the interval of $-\pi < x < \pi$, and using the appropriate test at suitable points, determine whether the given function is even, odd, or neither.

(5 marks)

(b) Prove that the corresponding Fourier series to this periodic function is given by:

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin[(2n-1)x]}{(2n-1)}, \qquad -\pi < x < \pi$$

Apply suitable equations as given in the appended formula.

(10 marks)

(c) By choosing an appropriate value for x, show that.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(5 marks)

Q6 (a) The heat flux through the faces at the ends of bar is found to be proportional to $u_n = \partial u/\partial n$ at the ends. If the bar is perfectly insulated and the ends $x = 0$ and $x = L$ are at adiabatic conditions,

$$u_x(0,t) = 0 \quad u_x(L,t) = 0$$

Prove that the solution of the heat transfer problem above (adiabatic conditions at both ends) is given as:

$$u(x,t) = A_0 + \sum_{n=1} A_n \cos\frac{n\pi x}{L} e^{-\left(\frac{\alpha n\pi}{L}\right)^2 t}$$

where A_o and A_n are arbitrary constants.
The heat equation is given as,

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

(15 marks)

(b) If $L = \pi$ and $\alpha = 1$ for the solution of heat transfer problem in **Q1(a)**, find the temperature in the bar with the initial temperature:

$$f(x) = k = constant$$

(5 marks)

- END OF QUESTIONS -

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION: SEM I / 2020-2021 PROGRAMME: BDD
COURSE NAME: DIFFERENTIAL EQUATIONS COURSE CODE: BDA 24303

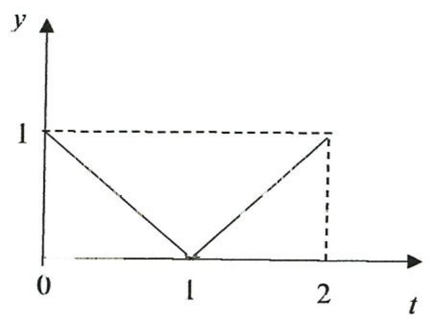

Figure Q3(a)

FINAL EXAMINATION

SEMESTER/SESSION: SEM I / 2020-2021 PROGRAMME: BDD
COURSE NAME: DIFFERENTIAL EQUATIONS COURSE CODE: BDA 24303

FORMULAS

First Order Differential Equations

Type of ODEs	General solution
Linear ODEs $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equations

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

Method of Undetermined Coefficients

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION: SEM I / 2020-2021 PROGRAMME: BDD
COURSE NAME: DIFFERENTIAL EQUATIONS COURSE CODE: BDA 24303

Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where

$$u_1=-\int\frac{y_2g(x)}{W}dx,$$

$$u_2=\int\frac{y_1g(x)}{W}dx,$$

where

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Laplace Transform

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION:	SEM I / 2020-2021	PROGRAMME:	BDD
COURSE NAME:	DIFFERENTIAL EQUATIONS	COURSE CODE:	BDA 24303

Laplace Transform (continued)

$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

Half Range Series

$$a_0=\frac{2}{L}\int_{0}^{L}f(x)dx$$

$$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

FINAL EXAMINATION

SEMESTER/SESSION: SEM I / 2020-2021 PROGRAMME: BDD
COURSE NAME: DIFFERENTIAL EQUATIONS COURSE CODE: BDA 24303

Trigonometric Identities

TANGENT IDENTITIES

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

RECIPROCAL IDENTITIES

$\csc\theta = \frac{1}{\sin\theta}$ $\sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

PYTHAGOREAN IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

PERIODIC IDENTITIES

$\sin(\theta + 2\pi n) = \sin\theta$

$\cos(\theta + 2\pi n) = \cos\theta$

$\tan(\theta + \pi n) = \tan\theta$

$\csc(\theta + 2\pi n) = \csc\theta$

$\sec(\theta + 2\pi n) = \sec\theta$

$\cot(\theta + \pi n) = \cot\theta$

EVEN/ODD IDENTITIES

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

DOUBLE ANGLE IDENTITIES

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

HALF ANGLE IDENTITIES

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

PRODUCT TO SUM IDENTITIES

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

SUM TO PRODUCT IDENTITIES

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

LAW OF TANGENTS

$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$

$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$

$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$

SUM/DIFFERENCES IDENTITIES

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$

MOLLWEIDE'S FORMULA

$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$

COFUNCTION IDENTITIES

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$

$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

LAW OF SINES

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$