UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

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DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

Q1 Consider a vector field $\mathbf{F}(x,y) = x^2\,\mathbf{i} - y\,\mathbf{j}$. As $(x,y) \to (0,0)$, $\|\mathbf{F}(x,y)\| \to 0$. True or false?

(1 mark)

(a) True
(b) False

Q2 What is the directional derivative of $f(x,y) = (1+xy)^{3/2}$ in the direction of the vector $\mathbf{a} = 4\,\mathbf{i} - 3\,\mathbf{j}$?

(2 marks)

(a) $\left(\frac{3}{2}x(1+xy)^{1/2}\,\mathbf{i} + \frac{3}{2}y(1+xy)^{1/2}\,\mathbf{j}\right)\cdot(4\,\mathbf{i}-3\,\mathbf{j})$
(b) $\left(\frac{3}{2}y(1+xy)^{1/2}\,\mathbf{i} + \frac{3}{2}x(1+xy)^{1/2}\,\mathbf{j}\right)\cdot(4\,\mathbf{i}-3\,\mathbf{j})$
(c) $\left(\frac{3}{2}x(1+xy)^{1/2}\,\mathbf{i} + \frac{3}{2}y(1+xy)^{1/2}\,\mathbf{j}\right)\cdot\left(\frac{4}{5}\,\mathbf{i}-\frac{3}{5}\,\mathbf{j}\right)$
(d) $\left(\frac{3}{2}y(1+xy)^{1/2}\,\mathbf{i} + \frac{3}{2}x(1+xy)^{1/2}\,\mathbf{j}\right)\cdot\left(\frac{4}{5}\,\mathbf{i}-\frac{3}{5}\,\mathbf{j}\right)$
(e) None of the above

Q3 Find the gradient of the function $f(x,y,z) = y\sin\left(\frac{x}{z}\right)$ at point $\left(\frac{\pi}{2},0,1\right)$.

(3 marks)

(a) $\mathbf{i}+\mathbf{j}$
(b) $\mathbf{i}$
(c) $\mathbf{j}$
(d) $\mathbf{k}$
(e) None of the above

Q4 Divergence is referred to the way the fluids flows either towards or away from a point. True or false?

(1 mark)

(a) True
(b) False

Q5 Find the divergence of the vector $\mathbf{F}(x,y,z) = xz^3\,\mathbf{i} + 2x^2y^4\,\mathbf{j} + 5yz^2\mathbf{k}$.

(3 marks)

(a) $z^3 + 8x^2y^3 + 10yz$
(b) $3xz^2 + 16xy^3 + 10z$
(c) $3z^2 + 8y^3 + 10z$
(d) $z^3 + 16xy^3 + 10y$
(e) None of the above

Q6 Compute the component of **i** in the curl of vector $\mathbf{F}(x,y,z) = xz^3\,\mathbf{i} + 2x^2y^4\,\mathbf{j} + 5yz^2\mathbf{k}$

(3 marks)

(a) $5z^2$
(b) $3xz^2$
(c) $4xy^4$
(d) $5z^2y$
(e) None of the above

Q7 Let C be the curve represented by the equations $x = t,\; y = 3t^2,\; z = 6t^3,\quad 0 \le t \le 1$. Evaluate the line integral, $\int_C xyz^2\,dx$ along C.

(3 marks)

Answer : ________________

Q8 Evaluate the integral $\int_C y\,dx - x\,dy$, where C is the upper half of the circle $x^2 + y^2 = 1$ from $(-1,0)$ to $(1,0)$.

(2 marks)

Answer : ________________

Q9 Find the work done by the force field $\mathbf{F}(x,y,z) = (y-z)\,\mathbf{i} + (x+z)\,\mathbf{j} + (x+y)\mathbf{k}$ on the particle that moves along a line segment from (1, 0, 0) to (3, 4, 2).

(2 marks)

Answer: ________________

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Q10 Find the integral form that deducted when using Green's Theorem to evaluate $\int_C x^2\,dx + (yx^3 + 3xy^3)\,dy$, where C is the boundary of the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

(2 marks)

(a) $\iint_D (3yx^2 + 3y^3)\,dA$

(b) $\iint_D (yx^3 + 3xy^3)\,dA$

(c) $\iint_D (x^2 - yx^3 + 3xy^3)\,dA$

(d) $\iint_D (2x - 3yx^2 + 3y^3)\,dA$

(e) None of the above

Q11 Given $\mathbf{F}(x, y, z) = 4x^3 y\,\mathbf{i} + 3y^4\,\mathbf{j} + 4z^3 y\,\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$
Find the integrand that derived when using Gauss's Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\,dS$.

(3 marks)

(d) $\iiint_G 12yx^2 + 12y^3 + 12yz^2 \; dV$

(e) $\iiint_G 4yx^3 + 3y^4 + 4yz^3 \; dV$

(f) $\iiint_G 4x^3 + 12y^3 + 4z^3 \; dV$

(d) $\iiint_G 12x^2 + 12y^3 + 12z^2 \; dV$

(e) None of the above

Q12 Compute the maximum rate of increase of function $x^2 + 4z^2 = 3y^2$ at point (2, 2, 1)

(2 marks)

(a) 14.967
(b) 4.899
(c) 11.867
(d) 6.767
(e) None of the above

Q13 Calculate the volume of the solid enclosed by the plane $2x+y+z=4$ in the first octant.

(2 marks)

(a) 16/3

(b) 16/5

(c) 31/3

(d) 31/12

(e) 31/6

Q14 Evaluate $\int_0^{\pi}\int_0^{x} x\sin y\,dydx$

(1 mark)

(a) $1+\frac{\pi^2}{2}$

(b) $2+\frac{\pi^2}{2}$

(c) $2+\frac{\pi}{2}$

(d) $1+\frac{\pi}{2}$

(e) $2+\pi^2$

Q15 By using double integral, find the volume of the solid enclosed by planes $x=y^2$ and $x+z=1$

(1 mark)

(a) $\int_0^1\int_{y^2}^1 1-xdydx$

(b) $\int_{-1}^1\int_{y^2}^1 1-xdydx$

(c) $\int_{-1}^1\int_1^{y^2} 1-xdxdy$

(d) $\int_0^1\int_1^{y^2} 1-xdxdy$

(e) $\int_0^1\int_{y^2}^1 1-xdxdy$

Q16 Changing the order of the integration in the double $\int_0^2 \int_0^{y^2} f(x,y)dxdy$ leads to $\int_q^p \int_s^r f(x,y)dydx$. What is p?

(2 mark)

(a) 4
(b) 2
(c) y^2
(d) $\sqrt{y}$
(e) 0

Q17 Convert $\int_0^1 \int_0^{\sqrt{1-x^2}} x - ydydx$ in polar coordinate

(1 mark)

(a) $\int_0^{\pi} \int_0^1 r\cos\theta - r\sin\theta drd\theta$
(b) $\int_0^{2\pi} \int_0^1 r^2\cos\theta - r^2\sin\theta drd\theta$
(c) $\int_0^{\frac{\pi}{2}} \int_0^1 r^2\cos\theta - r^2\sin\theta drd\theta$
(d) $\int_0^{\pi} \int_0^1 r^2\cos\theta - r^2\sin\theta drd\theta$
(e) $\int_0^{\pi} \int_0^1 r^2\sin\theta - r^2\cos\theta drd\theta$

Q18 Solve $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 4 - xdydx$

(2 marks)

(a) 26π
(b) 32π
(c) 18π
(d) 36π
(e) 9π

Q19 Evaluate $\iint_R \frac{1}{\sqrt{x^2+y^2}} dA$ where R is the region inside the circle $x^2 + y^2 = 2x$ and the outside the circle $x^2 + y^2 = 1$ in the first quadrant

(2 marks)

(a) $\sqrt{3} - \pi$
(b) $\sqrt{3} - \frac{\pi}{2}$
(c) $\sqrt{3} - \frac{\pi}{3}$

(d) $\sqrt{3}-\frac{\pi}{5}$

(e) $\sqrt{3}-\frac{\pi}{4}$

Q20 Find the surface area of the portion of hemisphere $z=\sqrt{25-x^2-y^2}$ that lies above the region R by the disc $x^2+y^2\leq 9$

(2 marks)

(a) 8π
(b) 10π
(c) 12π
(d) 14π
(e) 16π

Q21 By changing to cylindrical coordinate in $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{0}^{\sqrt{16-x^2-y^2}} zdzdydx$ is given by:

(1 mark)

(a) $\int_0^{2\pi}\int_0^{4}\int_0^{\sqrt{16-r^2}} zdzrdrd\theta$

(b) $\int_0^{\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} zdzrdrd\theta$

(c) $\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} rzdzdrd\theta$

(d) $\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} zdzdrd\theta$

(e) $\int_0^{\pi}\int_0^{2}\int_0^{\sqrt{16-r^2}} zdzdrd\theta$

Q22 Find the volume of the solid bounded by $y=x^2+z^2$ and the plane $y=9$.

(2 marks)

(a) $\frac{81\pi}{2}$

(b) $\frac{81\pi}{4}$

(c) $\frac{81\pi}{8}$

(d) $\frac{81\pi}{5}$

(e) $\frac{81\pi}{16}$

Q23 By changing to spherical coordinate, evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\,dzdydx$

(2 marks)

(a) $\frac{16\pi}{3}$

(b) $\frac{32\pi}{3}$

(c) $\frac{32\pi}{9}$

(d) $\frac{16\pi}{9}$

(e) $\frac{9\pi}{16}$

Q24 Find the volume of the solid bounded above by sphere $x^2+y^2+z^2=16$ and below by cone $z=\sqrt{x^2+y^2}$

(2 marks)

(a) $\frac{64\pi}{3}\left(1-\sqrt{2}\right)$

(b) $\frac{125\pi}{3}\left(1-\sqrt{2}\right)$

(c) $\frac{64\pi}{3}\left(2-\sqrt{2}\right)$

(d) $\frac{250\pi}{3}\left(2-\sqrt{2}\right)$

(e) $\frac{125\pi}{3}\left(2-\sqrt{2}\right)$

Q25 $\iint_R 1-x^2-y^2\,dA$, where R is the region enclosed between y-axis and the left half of the circle $x^2+y^2=5$ is given by:

(1 mark)

(a) $\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{5}} r-r^3\,drd\theta$

(b) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{5} r-r^3\,drd\theta$

(c) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{\sqrt{5}} r - r^3 \, dr d\theta$

(d) $\int_0^{\frac{3\pi}{2}} \int_0^{\sqrt{5}} r - r^3 \, dr d\theta$

(e) $\int_{\pi}^{2\pi} \int_0^{\sqrt{5}} \left(r - r^3\right) r dr d\theta$

Q26 Given $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz dy dx$. By changing to spherical coordinate, what is the value of θ ?

(1 mark)

(a) $\frac{\pi}{2} \leq \theta \leq 0$

(b) $\frac{\pi}{4} \leq \theta \leq 0$

(c) $\pi < \theta < 0$

(d) $2\pi \leq \theta \leq 0$

(e) $\frac{3\pi}{2} \leq \theta \leq 0$

Q27 Find the mass of a lamina with density $\delta(x, y) = 1 + x$ enclosed by x-axis, line x=1 and curve $y = \sqrt{x}$

(2 marks)

(a) $\frac{13}{20}$

(b) $\frac{15}{16}$

(c) $1\frac{1}{15}$

(d) $1\frac{2}{15}$

(e) $\frac{13}{16}$

Q28 Find the centroid of the lamina enclosed by $\sqrt{4-x^2}$ and above the x-axis.

(2 marks)

(a) $\left(0,\dfrac{8}{3\pi}\right)$

(b) $\left(0,\dfrac{4}{3\pi}\right)$

(c) $\left(\dfrac{8}{3\pi},0\right)$

(d) $\left(0,\dfrac{2}{3\pi}\right)$

(e) $\left(\dfrac{4}{3\pi},0\right)$

Q29 Find the moment of inertia I_x of the lamina that occupies the region $D=\{(x,y)0\le x\le 1, 0\le y\le 1\}$ and has the density function $\delta(x,y)=x$

(2marks)

(a) 1/24
(b) 1/18
(c) 1/12
(d) 1/6
(e) 1/4

Q30 Given $\int_0^3\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}}\sqrt{x^2+y^2+z^2}\,dzdydx$. By changing to spherical coordinate, what is the value of ϕ ?

(1 marks)

(a) $\dfrac{\pi}{2}\le\theta\le 0$

(b) $\dfrac{\pi}{4}\le\theta\le 0$

(c) $\pi\le\theta\le 0$

(d) $2\pi\le\theta\le 0$

(e) $\dfrac{3\pi}{2}\le\theta\le 0$

Q31 Find $\iiint_G x^2 y\,dV$, where the G is the solid bounded by $y = x^2$ and the planes $z = 0$, $y = 1$ and $z = y$.

(2 marks)

Short answer:

Q32 Given the solid that has density $\delta(x, y, z) = yz$ and is enclosed by $z = 1 - y^2$, $y = 0$, $z = 0$, $x = -1$ and $x = 1$.

a) Determine the center of gravity of the solid

(2 marks)

Short answer: ____________________

b) Analyze the centre of mass of lamina with density $\delta(x, y) = x + y$ that occupies the region bounded by $x = 0, y = 0$ and $y = 1 - x$.

(2 marks)

Short answer: ________________________________

Q33 Determine which of following paths to fulfill $\lim_{(x,y)\to(0,0)} \dfrac{12x^3 y}{4x^6 + 2y^2} = 2$

(2 marks)

(a) $y = x^2$
(b) $y = x^3$
(c) $x = y^3$
(d) $x = y^2$
(e) $x = y$

Q34 Determine the $\lim_{(x,y)\to(0,0)} \dfrac{xy}{2x^2 + y^2}$ along any linear $y = mx$

(2 marks)

(a) $\dfrac{m^2}{m^2+1}$
(b) $\dfrac{m}{m+2}$
(c) $\dfrac{2m}{m^2+1}$
(d) $\dfrac{m}{m^2+2}$
(e) $\dfrac{m}{2m^2+2}$

Q35 Find the equation of the plane tangent to the surface $f(x,y)=3x^2-y^2+2y$ at (1, -1, -1).

(2 marks)

(a) $z=6x+4y-1$

(b) $6x+4y+z=1$

(c) $4x+6y-z=-1$

(d) $z=-6x+2y+1$

(e) $6x-4y+z=2$

Q36 A cone has radius 3 cm and height 5 cm, respectively with the possible error 1% in each. Use partial derivatives to estimate the maximum possible error in calculating volume of the cone. Which of following steps are correct to calculate maximum possible error of volume of cone?

(2 marks)

(a) $dV=\frac{2}{3}\pi(3)(5)(0.3)+\frac{1}{3}\pi(3)(0.1)$

(b) $dV=\frac{1}{3}\pi(3)(5)(0.01)+\frac{2}{3}\pi(3)^2(0.01)$

(c) $dV=\frac{1}{3}\pi(3)(5)(0.03)+\frac{2}{3}\pi(3)^2(0.05)$

(d) $dV=\frac{2}{3}\pi(3)(5)(0.03)+\frac{1}{3}\pi(3)^2(0.05)$

(e) $dV=\frac{2}{3}\pi(3)(5)(0.01)+\frac{1}{3}\pi(3)^2(0.01)$

Q37 A solid cylinder of radius r and height h is heated. Calculate the increment in volume V, if r and h increase from 4.0 to 4.03 and from 10.0 to 10.2, respectively.

(2 marks)

(a) $\frac{28\pi}{3}$

(b) $\frac{28}{5}\pi$

(c) $\frac{26}{5}\pi$

(d) $\frac{27}{5}\pi$

(e) $\frac{26}{3}\pi$

Q38 The function $f(x,y) = x^2 + xy + y^2 - 3x$ has one critical points. Determine its location and type.

(2 marks)

(a) (-2, -1), minima points
(b) (2, -1), maxima points
(c) (-2, 1), maximum points
(d) (2, -1), minima points
(e) (-2, 1), saddle points

Q39 The total surface area of a cone of base radius r and perpendicular height, h is given by $S = \pi r\sqrt{r^2 + h^2} + \pi r^2$. If r and h are each increasing at the rate of 0.25 cm/s, find the rate at which the S is increasing at the instant when r=3cm and h=4cm.

(2 marks)

(a) 3.80π
(b) 3.08π
(c) 3.04π
(d) 3.40π
(e) 3.84π

Q40 Given $z = \sqrt{x^2 + y^2 + 6}$ Calculate the approximate change in z if (x, y) changes from $(1,1)$ to $(0.98, 1.02)$.

(2 marks)

(a) $\frac{2}{\sqrt{8}}(-0.02) + \frac{1}{\sqrt{8}}(0.02)$

(b) $\frac{1}{\sqrt{8}}(0.02) + \frac{1}{\sqrt{8}}(-0.02)$

(c) $\frac{1}{2\sqrt{2}}(-0.02) + \frac{1}{2\sqrt{2}}(0.02)$

(d) $\frac{1}{\sqrt{8}}(-0.02) + \frac{1}{\sqrt{8}}(-0.02)$

(e) None of the above

Q41 What is the domain for $\cos\frac{x}{3}$?

(2 marks)

(a) $x = \{x : -2\pi \le x \le 2\pi,\ x \in \Re\}$
(b) $y = \{y : -1 \le y \le 1,\ y \in \Re\}$
(c) $x = \{x : x \in \Re\}$
(d) $y = \{y : y \in \Re\}$
(e) None of the above

Q42 Let $f(x,y) = \ln(1 - 2x^2 - 3y^2)$. Find the domain for f.

(2 marks)

(a) $D = \{(x,y) : 2x^2 + 3y^2 > 1,\ x \in \Re, y \in \Re\}$
(b) $D = \{(x,y) : 2x^2 + 3y^2 < 1,\ x \in \Re, y \in \Re\}$
(c) $D = \{(x,y) : 1 - 2x^2 - 3y^2 < 1,\ x \in \Re, y \in \Re\}$
(d) $D = \{(x,y) : 1 - 2x^2 - 3y^2 \ge 0,\ x \in \Re, y \in \Re\}$
(e) None of the above

Q43 Let $f(x,y) = \ln(1 - 2x^2 - 3y^2)$. Find the range for f.

(2 marks)

(a) $R = \{z : z \ge 0, z \in \Re\}$
(b) $R = \{z : z \le 0, z \in \Re\}$
(c) $R = \{z : z \ge 1, z \in \Re\}$
(d) $R = \{z : z < 1, z \in \Re\}$
(e) None of the above

Q44 Let $f(x,y) = \dfrac{1}{3x - y^2}$. Find the domain for f.

(2 marks)

(a) $D = \{(x,y) : y^2 \ne 3x,\ x \in \Re, y \in \Re\}$
(b) $D = \{(x,y) : 3x - y^2 < 0,\ x \in \Re, y \in \Re\}$
(c) $D = \{(x,y) : 3x - y^2 < 1,\ x \in \Re, y \in \Re\}$
(d) $D = \{(x,y) : 3x^2 - y^2 \ne 0,\ x \in \Re, y \in \Re\}$
(e) None of the above

Q45 Let $f(x,y)=\dfrac{1}{3x-y^2}$. Find the range for f.

(2 marks)

(a) $R=\{z : z \geq 0, z \in \Re\}$
(b) $R=\{z : z \leq 0, z \in \Re\}$
(c) $R=\{z : z \neq 0, z \in \Re\}$
(d) $R=\{z : z \geq 1, z \in \Re\}$
(e) None of the above

Q46 If $z=\frac{x^2-2xy^2+y^3}{x^2-2xy^2}$, *what is* f_{xy}- f_{yx} at point (1,1,1)

(2 marks)

Answer: ____________________

Q47 Evaluate f_{yy} for function $f(x,y)=(2x^2-y)^2$

(2 marks)

Answer: ____________________

Q48 Given $w=\frac{\sqrt{xy+y}}{z}, x=\cos\theta, y=\sin\theta, z=\theta$, find $\frac{dw}{d\theta}$ when $\theta=\frac{\pi}{2}$

(2 marks)

Answer: ____________________

Q49 Describe the graph of the function $f(x,y)=3+x^2+y^2$.

(2 marks)

Answer: ____________________

Q50 What is the 3D graph of $x^2+y^2+4y=0$?

(2 marks)

(a) A cylinder with radius 4 units, parallel to z – axis.
(b) A paraboloid with the vertex (4, 0), parallel to z – axis.
(c) A cylinder with radius 2 units, parallel to z – axis.
(d) A sphere with radius 2 units.
(e) None of the above

-END OF QUESTIONS-