UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(TAKE HOME)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEJ 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JULY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **CLOSED BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **SIXTEEN (16)** PAGES

Q1. (a) Let $x(t)$ and $y(t)$ given in **Figure Q1(a)** and **Figure Q1(b)**, respectively. Sketch the following signals.

(i) $x(t+1)y(t-2)$

(3 marks)

(ii) $x(4-t)y(2t)$

(3 marks)

(b) Sketch the following signals

(i) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

(3 marks)

(ii) $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

(3 marks)

(c) **Figure Q1(c)** shows $x(t)$ that may be viewed as the superposition of ***THREE* (3)** signals. Starting with the rectangular pulse $g(t)$of **Figure Q1(d)**.

(i) Construct **THREE (3)** signals for these superposition using transformation of $g(t)$.

(6 marks)

(ii) Express $x(t)$ in terms of $g(t)$.

(2 marks)

Q2. (a) Test the stability of the following continuous-time systems.

(i) $h_1(t) = e^t u(t)$

(3 marks)

(ii) $h_2(t) = e^{-2t} u(t+4)$

(3 marks)

(b) An LTI system with an input response, $x(t)$ resulting a system response, $y(t)$ as per shown in **Figure Q2(b)(i)**. Given the input response, $x(t)$ as in **Figure Q2(b)(ii)**.

(i) Find the convolution integral of the LTI system if the $h(t)$ is given as in **Figure Q2(b)(iii)**.
Hint: Do not have to sketch the $y(t)$ graph for your answer.

(12 marks)

(ii) Given the signal $h_1(t) = u(t+1) - u(t)$. By referring to the **Figure Q2(b)(i)**, find the signal of $h_2(t)$.

(2 marks)

Q3 (a) A voltage source $v_{in}(t)$ is expressed in complex exponential Fourier series and is given by:

$$v_{in}(t) = 10 + \sum_{\substack{n=3 \\ n \neq 0}}^{3} j \frac{1}{n\pi} e^{(jn\pi(50k)t)}$$

(i) Draw the magnitude and phase spectrum of $v_{in}(t)$.

(5 marks)

(ii) Express the output of the system, $v_{out}(t)$ if signal $v_{in}(t)$ in **Q3(a)(i)** becomes the input to an LTI system as shown in **Figure Q3(a)**.

(8 marks)

(b) Given signal $v_s(t)$ become the input to the circuit as shown in **Figure Q3(b)**.

$$v_s(t) - 2 + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{\pi} \sin(n\pi t)$$

Determine the current of the system $i(t)$.

(7 marks)

Q4 (a) **Figure Q4(a)** shows two systems which are interconnected in parallel. Determine the Fourier transform of the signals below by using Fourier transform properties:

(i) $P(\omega)$

(1 mark)

(ii) $Q(\omega)$

(1 mark)

(iii) $R(\omega)$

(2 marks)

(iv) $S(\omega)$

(2 marks)

(v) $Y(\omega)$

(2 marks)

(b) A cascaded system that consists of an LTI system and a delay system is shown in **Figure Q4(b).** The input signal $x(t)$ and impulse response of the LTI system, $h(t)$ are given as the following:

$$x(t) = e^{-2t}u(t)$$
$$h(t) = e^{\ t}u(t)$$

Determine:

(i) The Fourier transform of $y(t)$.

(3 marks)

(ii) The Fourier transform of $z(t)$.

(3 marks)

(c) A basic modulator circuit is shown in **Figure Q4(c).** Modulation is a multiplication between input signal, $m(t)$, and a carrier signal, $c(t)$. The process yields a modulated signal, $y(t)$. Frequency of carrier signal is higher than frequency of the input signal ($\omega_c > \omega_m$).

(i) Analyze the Fourier Transform of signal $y(t)$ by using modulation properties.

(4 marks)

(ii) Sketch the amplitude spectrum signal of $Y(\omega)$.

(2 marks)

Q5 (a) Given a signal $x(t) = 2\cos(2\pi t)\,u(t)$. Find the Laplace transform of $x(t)$ using the definition of Laplace transform and its region of convergence (ROC)

(4 marks)

(b) The Laplace transform of a signal $x(t)$ is

$$\mathcal{L}[x(t)] = X(s) = \frac{1}{s+1}, ROC > \ 1$$

Using the properties of Laplace transform, determine the Laplace transform of

$$\frac{d}{dt}\left[x\left(\frac{t-1}{2}\right)\right]$$

(8 marks)

(c) **Figure Q5(c)** shows an LTI system with impulse response of $h(t) = 2e^{-2t}u(t)$. The input signal of the LTI system is $x(t) = u(t)$. Determine:

(i) The system causality.

(2 marks)

(ii) The system stability.

(2 marks)

(iii) The Laplace transform of $y(t)$.

(4 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

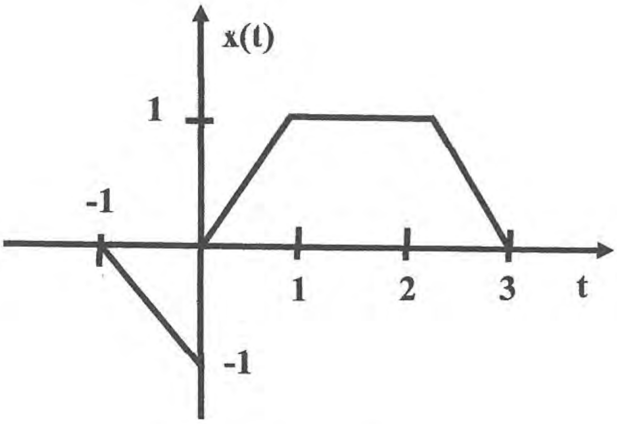

Figure Q1(a)

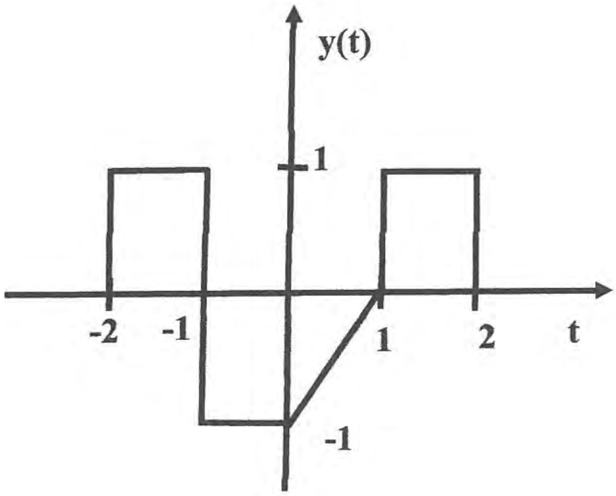

Figure Q1(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ

COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

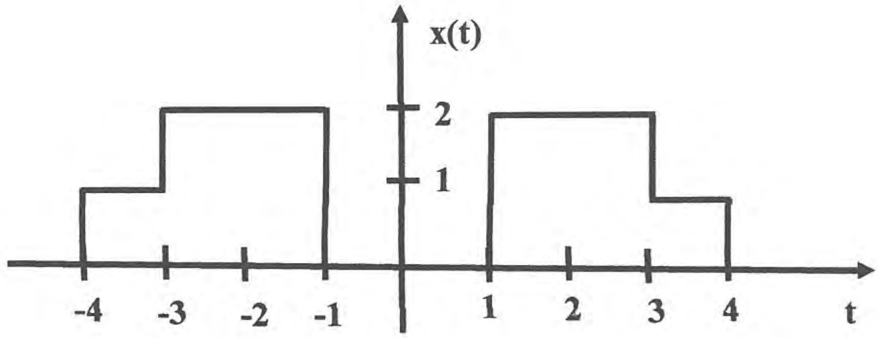

Figure Q1(c)

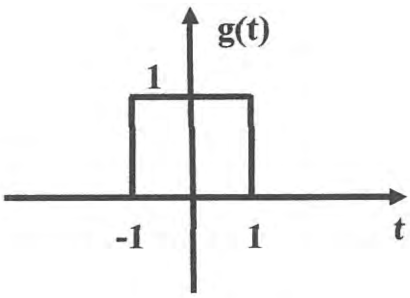

Figure Q1(d)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

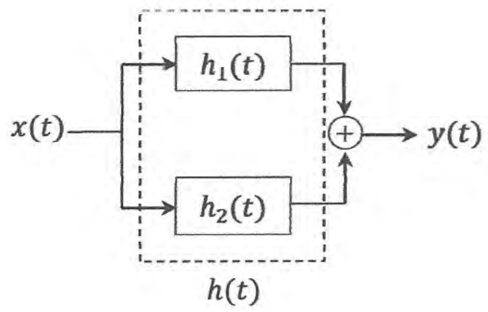

Figure Q2(b)(i)

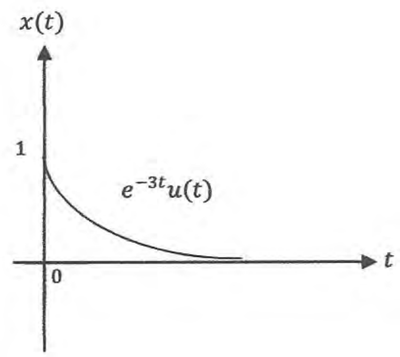

Figure Q2(b)(ii)

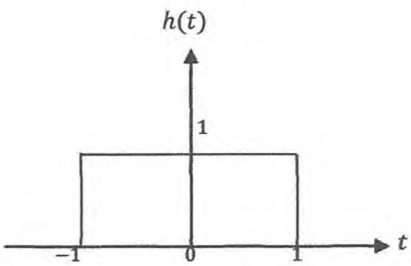

Figure Q2(b)(iii)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

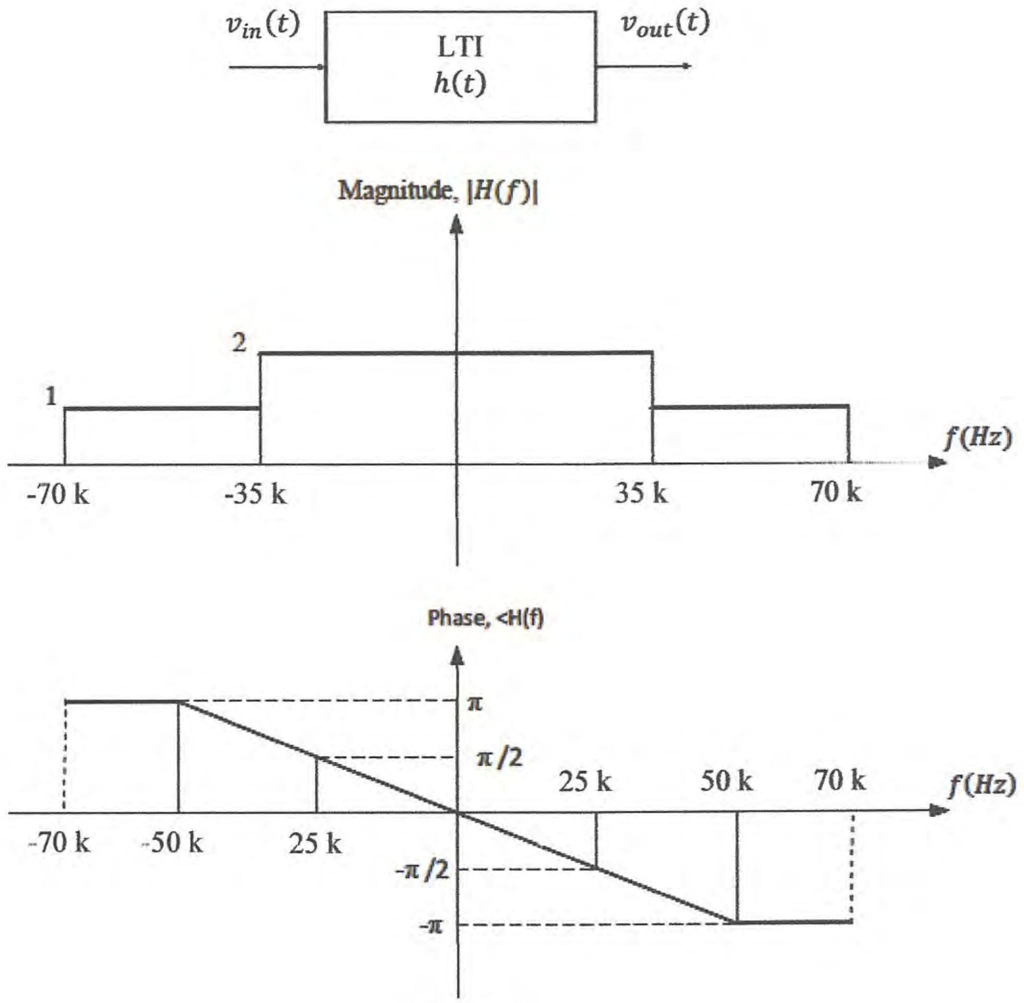

Figure Q3(a)

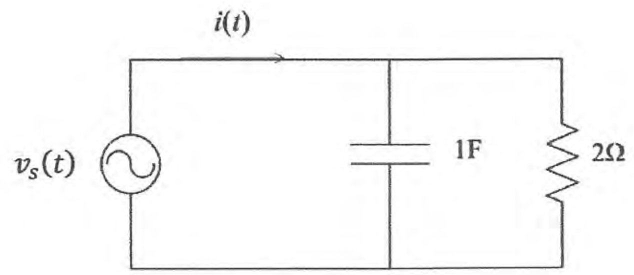

Figure Q3(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 | PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS | COURSE CODE: BEJ 20203

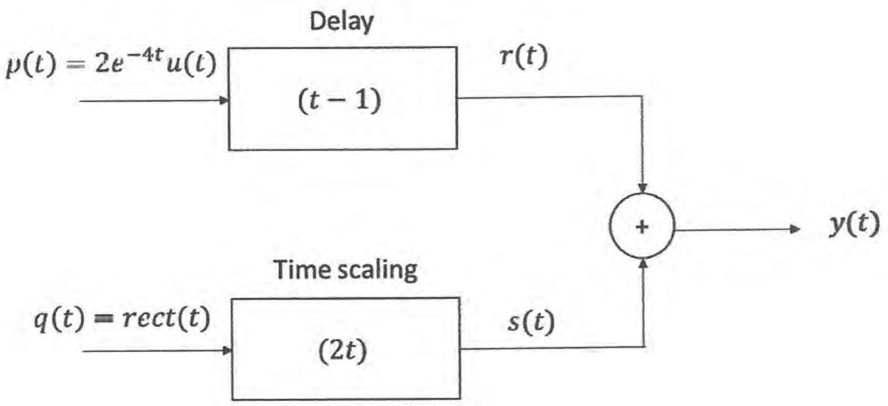

Figure Q4(a)

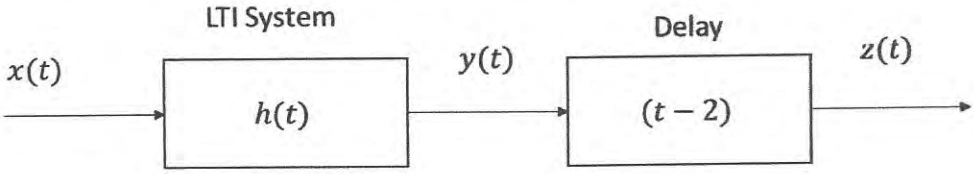

Figure Q4(b)

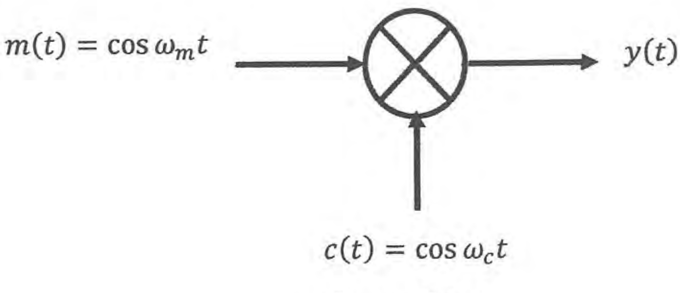

Figure Q4(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

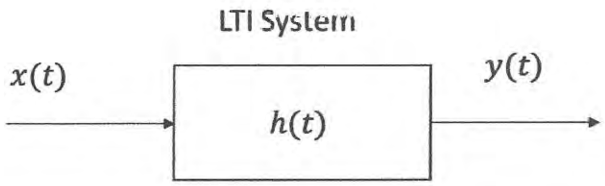

Figure Q5(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ

COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\,dt = \frac{1}{a}\sin at$	$\int \sin at\,dt = -\frac{1}{a}\cos at$
$\int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} - \pm j$	$A\angle \pm \theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm}j\theta = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \lvert s\rvert\angle \pm \theta = \lvert s\rvert e^{\pm j\theta}$	$\lvert s\rvert = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right))$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even} \\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd}\end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even} \\ 0 & ,n = \text{odd}\end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n-1}{2}} & ,n = \text{odd} \\ 0 & ,n = \text{even}\end{cases}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

TABLE 6: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t} dt$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \cos n\frac{2\pi}{T}t\, dt, \quad n = 0, 1, 2, 3 \ldots$ $b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \sin n\frac{2\pi}{T}t\, dt, \quad n = 1, 2, 3 \ldots$
Amplitude-phase	$x(t) = X_0 + \sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t + \theta_n)$ $A_n = 2\lvert X_n\rvert = \sqrt[2]{a_n^2 + b_n^2} \quad , \quad \theta_n = \angle X_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

TABLE 7: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\, dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}\, df$
LAPLACE TRANSFORM	**INVERSE LAPLACE TRANSFORM**
Bilateral $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	$x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

TABLE 8: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$
$u(t+\tau) - u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega} = 2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\omega)$	$\mathrm{sinc}(f)$
$\lvert t\rvert$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{a+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{a-j2\pi f}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2 f^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

TABLE 9: FOURIER TRANSFORM PROPERTIES

Property	Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$	$a_1X_1(f)+a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{j}{2}[X(\omega+\omega_0)-X(\omega-\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{j}{2}[X(f+f_o)-X(f-f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$	$j2\pi f X(f)$ $(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^n x(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)$ or $X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	$X(f)*Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2df$

FINAL EXAMINATION

SEMESTER/SESSION: SEM II/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEJ 20203

TABLE 10: LAPLACE TRANSFORM PAIR

Time domain, $x(t),\ t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t),\ t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 11: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)\cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$