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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(TAKE HOME)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 21503
PROGRAMME	:	BEV / BEJ
EXAMINATION DATE	:	JULY 2020
DURATION	:	6 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) If $w = x^2 - 2y^2 + z^3$, $x = \sin t$, $y = e^t$, $z = 3t$, use Chain Rule to find $\frac{dw}{dt}$.

(8 marks)

(b) Show that the function $z = \ln\sqrt{x^2 + y^2}$ satisfies equation

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$$

(7 marks)

(c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = f(x, y)$ is implicitly defined as a function of x and y for equations:

(i) $z^2 + z\sin(xy^2) = 0$

(ii) $ye^{xz} - 3ye^{yz} = -4ze^{xy} + 1$.

(10 marks)

Q2 (a) Given the double integral

$$\iint_R xy\, dA$$

where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and $x^2 + y^2 = 25$,

(i) Sketch the region bounded by the equations for the first quadrant

(2 marks)

(ii) Evaluate the double integral by using suitable coordinate system based on the interval from the region in **Q2(a)(i)**.

(9 marks)

(b) Given the triple integrals

$$\iiint_E \sqrt{x^2 + y^2}\, dV$$

where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

(i) Sketch the cylinder and its projection on suitable plane.

(4 marks)

(ii) Evaluate the triple integrals using any suitable coordinate system for the boundary conditions in **Q2(b)**(i)

(10 marks)

Q3 (a) Compute the work done of a force, $\mathbf{F}(x, y) = 2x\mathbf{i} + y^2\mathbf{j}$ in order to move a particle from point (0, 0) to (3, 9) along the following curves:

(i) C_1: y = 3x

(ii) C_2: $y - x^2$

(9 marks)

(b) Find the potential function of **F**.

(9 marks)

(c) Based on the potential function in **Q3(b),** verify the work done calculated in **Q3(a)** and justify why the calculated work done are the same.

(2 marks)

(d) Compute the surface integral $\iint_\sigma dS$, where σ is the first-octant portion of the plane $x + y + z = 9$.

(5 marks)

Q4 (a) Let the velocity field is given by

$$\mathbf{F} = xy\,\mathbf{i} - \frac{y^2}{2}\mathbf{j} + z\,\mathbf{k}$$

and σ is the surface of the solid G enclosed by paraboloid $z = 4 - 3x^2 - 3y^2$ and plane $z = 1$ oriented outward.

(i) Compute the flux of water flowing through the surface of the paraboloid across the velocity field **without** using Gauss's theorem.

(9 marks)

(ii) Evaluate $\iint_\sigma \vec{F} \cdot \hat{n} ds$ by using Gauss's theorem.

(5 marks)

(b) Let the force field is given by

$$\mathbf{F} = \left(\sin x - \frac{y^3}{3}\right)\mathbf{i} + \left(\cos y + \frac{x^3}{3}\right)\mathbf{j} + xyz\,\mathbf{k}$$

and σ be the portion of upper cone, $z = \sqrt{x^2 + y^2}$, intersects with the plane $z = 1$. Suppose that the curve C is the boundary of σ with the plane $z = 1$ oriented anticlockwise.

(i) Find the work done by the force field along the curve C.

(4 marks)

(ii) Verify the Stokes' theorem.

(7 marks)

END OF QUESTIONS

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FORMULAS

Polar coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\,dr\,d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z) r\,dz\,dr\,d\theta$

Spherical coordinate

$x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi$, then $x^2+y^2+z^2=\rho^2$, for $0\le\theta\le 2\pi,\ 0\le\phi\le\pi$,

and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$A = \iint_R dA$

$m = \iint_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina

$V = \iint_R f(x,y)\,dA$

$V = \iiint_G dV$

$m = \iiint_G \delta(x,y,z)dV$

If f is a differentiable function of x, y and z, then the

Gradient of f, grad $f(x,y,z) = \nabla f(x,y,z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

If $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is a vector field in Cartesian coordinate, then the

Divergence of $\mathbf{F}(x,y,z)$, div $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$ **Curl of** $\mathbf{F}(x,y,z)$,

$$\text{curl } \mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

$\mathbf{F}$ is conservative vector field if Curl of $\mathbf{F} = 0$.

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Surface Integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$$

Stokes' Theorem

$$\iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS = \oint_C \mathbf{F}\bullet d\mathbf{r}$$

Identities of Trigonometry and Hyperbolic

Trigonometric Functions

$$\cos^2 x+\sin^2 x=1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x-\sin^2 x$$
$$= 2\cos^2 x-1$$
$$= 1-2\sin^2 x$$
$$1+\tan^2 x = \sec^2 x$$
$$1+\cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x\pm y) = \frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$
$$\sin(x\pm y) = \sin x\cos y\pm\cos x\sin y$$
$$\cos(x+y) = \cos x\cos y\mp\sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x+\sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x-\cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x+\cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x-e^{-x}}{2}$$
$$\cosh x = \frac{e^x+e^{-x}}{2}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x+\sinh^2 x$$
$$= 2\cosh^2 x-1$$
$$= 1+2\sinh^2 x$$
$$1-\tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x-1 = \operatorname{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x\pm y) = \frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$$
$$\sinh(x\pm y) = \sinh x\cosh y\pm\cosh x\sinh y$$
$$\cosh(x\pm y) = \cosh x\cosh y\pm\sinh x\sinh y$$

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The derivative of f(x) with respect to x

$$f_x(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Indefinite Integrals and Integration of Inverse Functions

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \text{sech}^2 x dx = \tanh x + C$$

$$\int \text{csch}^2 x dx = -\coth x + C$$

$$\int \text{sech} x \tanh x dx = -\text{sech} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 < a^2$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 < a^2$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{a^2 + x^2} dx = \cot^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{|x|\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 > a^2$$

$$\int \frac{-1}{|x|\sqrt{x^2 - a^2}} dx = \frac{1}{a} \csc^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 > a^2$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a > 0$$

$$\int \frac{-1}{|x|\sqrt{a^2 - x^2}} dx = \frac{1}{a} \text{sech}^{-1}\left|\frac{x}{a}\right| + C, \quad 0 < x < a$$

$$\int \frac{-1}{|x|\sqrt{a^2 + x^2}} dx = \frac{1}{a} \text{csch}^{-1}\left|\frac{x}{a}\right| + C, \quad x \neq 0$$

$$\int \frac{1}{a^2 - x^2} dx = \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C, & x^2 < a^2 \\ \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + C, & x^2 > a^2 \end{cases}$$