CONFIDENTIAL

UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2020
DURATION	:	6 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

CONFIDENTIAL

Q1 (a) Find the particular solution for $y(x) = Ax^2 + \frac{B}{x^3}$ with boundary value $y(2) = 1$ and $y'(1) = 0$.

(4 marks)

(b) Find the particular solution of the differential equation $\frac{dy}{dx} - y = e^{2x}$ which satisfies the initial condition $y = 3$ when $x = 0$ using the linear equation method.

(7 marks)

(c) Tank contains 50 g of salt dissolved in 100 L of water. Tank capacity is 400 L. At time t = 0, 0.25 g/L of salt is entering at a rate of 4 L/min, and the well-mixed mixture is drained at 2 L/min. Calculate:

(i) Time, t when it overflows

(3 marks)

(ii) Amount of salt before overflow.

(9 marks)

Q2 (a) By using the suitable substitution, proof that $\theta = 4t - \frac{1-9e^{-4t}}{1+3e^{-4t}}$ is a particular solution of differential equation $\frac{d\theta}{dt} = (4t - \theta + 1)^2$. Hence, satisfy the initial condition $\theta(0) = 2$.

(10 marks)

(b) Determine the general solution for $2y'' - 6y' - 36y = 4e^{4x}$

(7 marks)

(c) A spring with length l m is attached to a weighing scale as in **Figure Q2 (c)**. An asphalt concrete sample is then attached to the end of the spring and caused the spring to elongate by a m and weighing scale showing the reading of 5 kg. Subsequently, the sample is submerged in water and is pulled down causing a spring displacement of y m. The sample is then accidentally released causing it to oscillates up and down in water. Develop the equation of the sample motion in water. Given elongation of $y = 0.05\ m$ spring constant k = 700 N/m and friction force of water = $\frac{1}{5}$ velocity and weight of spring is negligible.

(12 marks)

Q3 (a) Find the inverse Laplace Transform of the given expressions:

(i) $\frac{100}{s}$

(2 marks)

(ii) $\frac{1}{s-50}$

(3 marks)

(b) Show that $\mathcal{L}\{t\cos 4t\} = \frac{s^2-16}{(s^2+16)^2}$

(7 marks)

(c) By using Laplace transform method, solve the initial value problem at $Q(0) = 0$ and $Q'(0) = 0$.

$$2\frac{d^2Q}{dt^2} + 16\frac{dQ}{dt} + \frac{Q}{0.02} = 300$$

(12 marks)

Q4 (a) Consider the series:

$$\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots\ldots\ldots\ldots$$

(i) Give the sigma notation of the above series

(2 marks)

(ii) Determine whether the series converge or diverge

(5 marks)

(b) A periodic function is defined by

$$f(x) = \begin{cases} 0, & -\pi < x < -\frac{\pi}{2}, \\ 4, & -\frac{\pi}{2} < x < \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} < x < \pi, \end{cases}$$

$f(x) = f(x + 2\pi)$.

(i) Sketch the graph of the function over $-2\pi < x < 2\pi$.

(4 marks)

(ii) Determine whether the function is even, odd or neither.

(2 marks)

(iii) Show that the Fourier series of the function $f(x)$ is

$$2 + \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{n}\cos nx$$

(6 marks)

(c) Determine the interval and radius of convergence for the following power series.

$$\sum_{n=0}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}$$

(5 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003

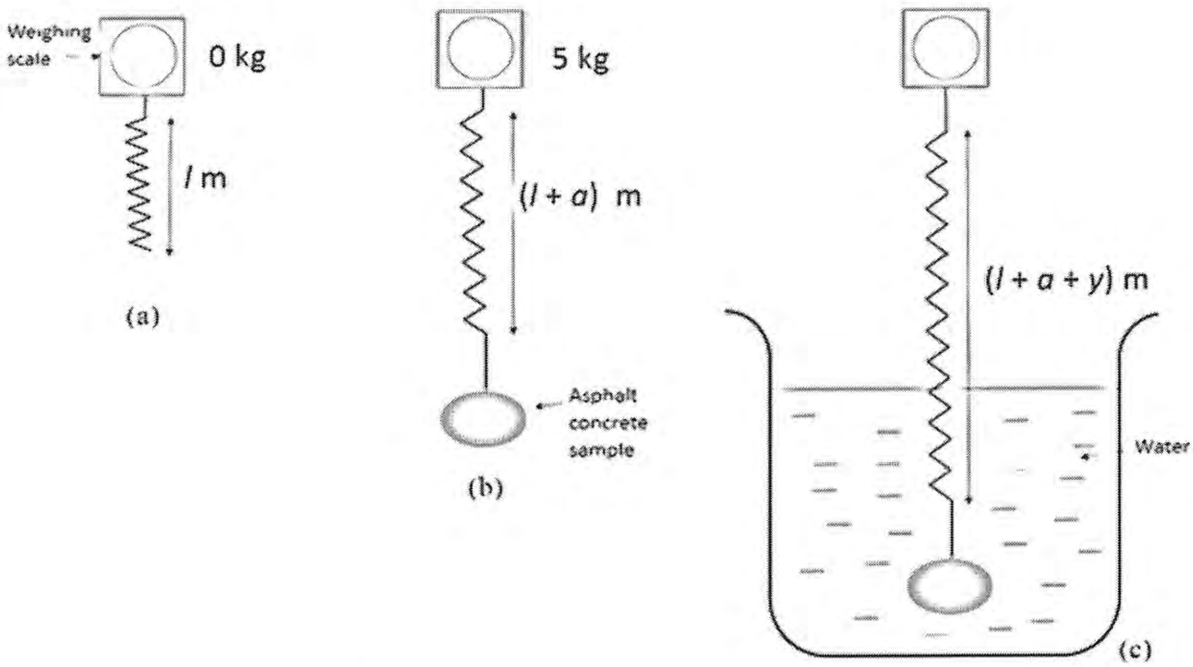

FIGURE Q2 (c): Illustration of spring condition

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003

Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx, \quad u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003

Laplace Transforms

$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$			
$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s > 0.$

TERBUKA

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003

Fourier Series

Fourier series expansion of periodic function with period $2L$	**Fourier half-range series expansion**
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$
where	where
$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$	$a_0=\frac{2}{L}\int_{0}^{L}f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003

Trigonometric and Hyperbolic Identities

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x - 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	

FINAL EXAMINATION

SEMESTER / SESSION: SEM II / 2019/2020
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II

PROGRAMME CODE: BFF
COURSE CODE: BFC 14003

Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x \cot x$	$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x \tanh x$	$\int \text{sech}\,x \tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x \coth x$	$\int \text{cosech}\,x \coth x\,dx = -\text{cosech}\,x + C$