Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS I
COURSE CODE	:	BFC13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2020
DURATION	:	6 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF NINE (9) PAGES

Q1 (a) A function h is defined by

$$\square(x) = \begin{cases} ax - 11 & x \leq -2 \\ bx^3 + c & -2 < x \leq 1 \\ 4 - x^2 & x > 1 \end{cases}$$

(i) Find the value of a if $\lim_{x \to -2} h(x) = -15$

(2 marks)

(ii) Determine the value of b and c if $\lim_{x \to -2} \square(x)$ and $\lim_{x \to 1} \square(x)$ exist.

(3 marks)

(b) Differentiate $y = cot\, x$ with respect to x by using suitable method.

(4 marks)

(c) The parametric equations of a curve are given by $x = 3t + 8$ and $y = t - 3t^2$. Find $\frac{dy}{dx}$ in terms of x.

(4 marks)

(d) Given the implicit function $xe^y + yln x = 5$, find $\frac{dy}{dx}$ in terms of x and y.

(7 marks)

Q2 (a) Find the set of values of x for which the graph of the function $f(x) = 2x^3 - 12x + 11$

(i) Is concave upwards.

(2 marks)

(ii) Is concave downwards.

(3 marks)

(b) The given **FIGURE Q2 (b)** shows a trapezium ABCD such that DA = AB = BC = 10cm and ∠DAP = ∠ CBQ = θ radians. Show that the area of the trapezium, L cm^2, is given by $L = cos\, \theta\, (1 + sin\, \theta)$. Examine the value of θ such that the area of the trapezium is a maximum and calculate the maximum area of the trapezium.

(15 marks)

Q3 (a) **FIGURE Q3 (a)** shows a combined solid which consists of a cuboid surmounted by a semi-cylinder with a common face PQRS. The breadth and the length of the cuboid are x cm and $2x$ cm respectively and its height is y cm. Given that the total surface area of this combined solid is 9600 cm^2, show that $y = \frac{1}{24x}\,[\,38400 - (8 + 5\pi)x^2]$. If the volume of the combined solid is V cm^3, show that $V = \frac{1}{6}\,[\,19200x - (4 + \pi)x^3]$. Find the value of x, in terms of π, when V is a maximum and show that V is a maximum. Calculate its maximum volume, stating your answer correct to the nearest integer.

(10 marks)

(b) On the same coordinate axes, sketch the graph of $y = e^x$ and $y = 3 + 4e^{-x}$. If the region bounded by the curves and the y-axis is revolved through 2π radians about the x-axis, show that the volume of the solid generated is $(9 \ln 4 + 18)\pi$ units^3.

(10 marks)

Q4 (a) On the same coordinate axes, sketch the graphs of the curve $y^2 = 9(x+1)$ and the straight line $y = -x + 9$. Hence, calculate the area of the region bouded by the curve, the staright line and y-axis.

(10 marks)

(b) On the same coordinate axes, sketch the graphd of hte curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the straight line $3y = -2x + 6$. Hence, analyze the volume of the solid generated when the region bouded by the curve and the straight line is rotated through 2π radians about the y-axis.

(10 marks)

Q5 (a) Differentiate $cot^{-1}\sqrt{x}$ respect to x

(4 marks)

(b) Evaluate $\int \frac{dx}{\sqrt{4x^2-2}}, x > \frac{3}{2}$

(4 marks)

(c) Express $\int \frac{x+1}{\sqrt{1-x^2}} dx$ in the form of $\frac{A\frac{d}{dx}(1-x^2)+B}{\sqrt{1-x^2}}$, where A and B are constants. Hence,

evaluate $\int_0^{1/2} \frac{x+1}{\sqrt{1-x^2}} dx$.

(12 marks)

– END OF QUESTIONS –

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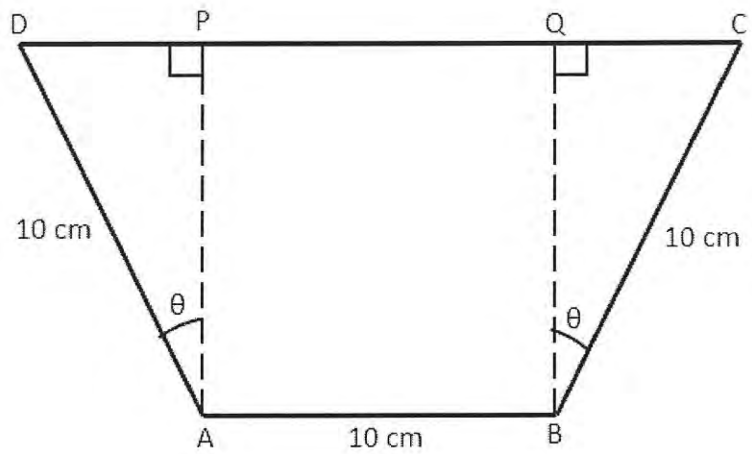

FIGURE Q2 (b)

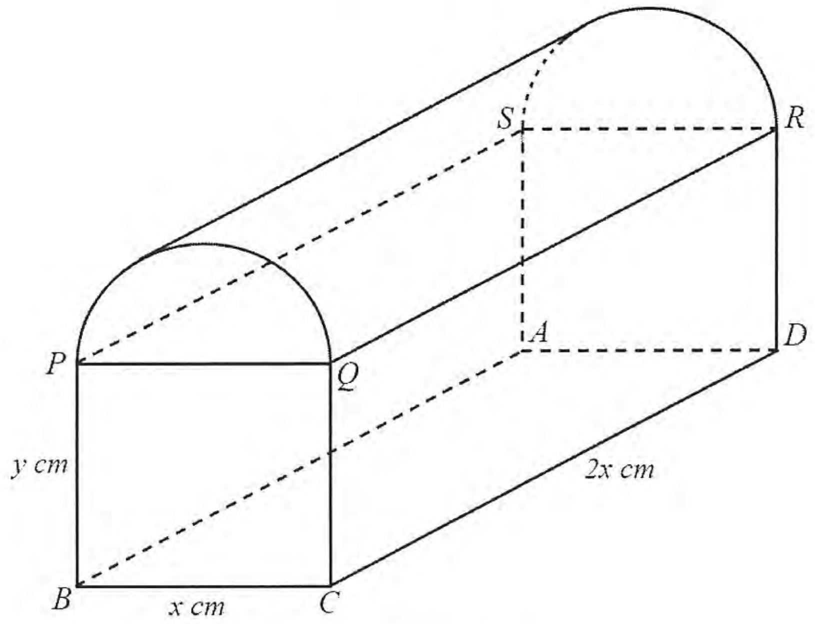

FIGURE Q3 (a)

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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\|+C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec } x]=-\text{cosec x} \cot x$	$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech } x]=-\text{sech } x \tanh x$	$\int \text{sech}\,x \tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech } x]=-\text{cosech } x \coth x$	$\int \text{cosech}\,x \coth x\,dx = -\text{cosech}\,x + C$

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Formulae

Trigonometric	**Hyperbolic**
$\cos^2 x+\sin^2 x=1$	$\sinh x=\dfrac{e^x-e^{-x}}{2}$
$1+\tan^2 x=\sec^2 x$	$\cosh x=\dfrac{e^x+e^{-x}}{2}$
$\cot^2 x+1=\csc^2 x$	$\cosh^2 x-\sinh^2 x=1$
$\sin 2x=2\sin x\cos x$	$1-\tanh^2 x=\operatorname{sech}^2 x$
$\cos 2x=\cos^2 x-\sin^2 x$	$\coth^2 x-1=\operatorname{csch}^2 x$
$\cos 2x=2\cos^2 x-1$	$\sinh 2x=2\sinh x\cosh x$
$\cos 2x=1-2\sin^2 x$	$\cosh 2x=\cosh^2 x+\sinh^2 x$
$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$	$\cosh 2x=2\cosh^2 x-1$
$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$	$\cosh 2x=1+2\sinh^2 x$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$
$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$
$2\sin x\cos y=\sin(x+y)+\sin(x-y)$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$2\sin x\sin y=-\cos(x+y)+\cos(x-y)$	$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1+\tanh x\tanh y}$
$2\cos x\cos y=\cos(x+y)+\cos(x-y)$	**Inverse Hyperbolic**
Logarithm	$\sinh^{-1} x=\ln\left(x+\sqrt{x^2+1}\right)$, any x
$a^x=e^{x\ln a}$	$\cosh^{-1} x=\ln\left(x+\sqrt{x^2-1}\right)$, $x\geq 1$
$\log_a x=\dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1<x<1$

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Formulae

Differentiation of Inverse Functions	
y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\tan^{-1} u$	$\frac{1}{1+u^2}\frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2}\frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\text{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\tanh^{-1} u$	$\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\coth^{-1} u$	$-\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\text{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}, \quad 0 < u < 1$
$\text{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}, \quad u \neq 0$

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Formulae

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\lvert a\rvert\sqrt{a^2-x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+C, \quad a>0$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert>a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$

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Formulae

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $K = \dfrac{\left[\frac{d^2y}{dx^2}\right]}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$

Curvature of parametric curve

Curvature, $K = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$