Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION (ONLINE) SEMESTER II SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC IV
COURSE CODE	:	BFC24203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2020
DURATION	:	6 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Given the function $f(x) = x^3 - 5x^2 - 2x + 10$.

(i) Prove that there at least a root in the interval [1,3] by using Intermediate Value Theorem.

(2 marks)

(ii) Find the root of $f(x)$ by using Bisection method. Iterate until $i = 5$.

(8 marks)

(b) (i) Prove the Lagrange interpolating polynomial of second degree for data of (0,1), (1,2) and (4,2) is $P2(x) = -\frac{1}{4}x^2 + \frac{5}{4}x + 1$.

(5 marks)

(ii) If then, one data points (5,3) are added to the previous data, find the new Lagrange interpolating polynomials without solving it.

(10 marks)

Q2 (a) **Table Q2(a)** tabulates the approximation values of function, *f(x)*. Complete the table for $f''(x_i)$ column in the range of $2.1 \leq x \leq 2.4$ using 3-point central difference formula. Then, calculate the error based on the exact value given by $f(x) = e^x$. Which x_i gives the best approximation?

(10 marks)

(b) **Figure Q2(b)** shows the dam retains 10m of water. A sheet pile wall (cut off curtain) on the upstream side, which is used to reduce seepage under the dam, penetrates 7m into a 20.3 m thick silty sand stratum below. The data for head pressure, h_p and the point interval distance, x is provided in **Table Q2(b).** From the data x and h_p, the pore water pressure, u can be calculated where $u = 9.81h_p$. Therefore, as an engineer you need to approximate the pore water pressure, u by applying appropriate Simpson's Rule. Calculate in three decimal places.

(15 marks)

Q3 (a) Assume v (0) = (1 1 0) ^{T}, iterate the matrix until the error value $|\varepsilon|<0.005$ by using power method. Find the dominant eigenvalue, $\lambda_{largest}$ in absolute value and show the eigenvector, v_1 of matrix A. Give your answer to three decimal places.

$$A=\begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

(10 marks)

(b) Given the total length of insulated metal bar, AB is 4m with taking $\Delta x = h = 1$. Point of A is kept at 0°C, while the other points to the end of B is maintained at 10°C until a steady state of temperature along the bar is achieved. At t= 0s, however the end of point B is suddenly reduced to 0°C while the other points are kept at the same temperature. Use the implicit method to solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial u^2}{\partial x^2}$ by taking $k = \Delta t = 0.2s$ until $t = 0.4s$ only.

(15 marks)

Q4 You are assigned as a design engineer and required to complete a project using Matlab software. You need to introduce the mesh point for the distance between 1 and 6 for a boundary value problem (BVP) of $y'' - xy' + 4y = 8x$. By substituting the finite difference approximation of $y'_i \approx \frac{y_{i+1} - y_{i-1}}{?h}, y''i \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$ into the equation, you need to solve the problem with given two mesh points $y(0) = 2$ and $y(5)$=25 and Δx=h=1.0. In report submission, you also need to discuss the importance of finite difference method in solving the boundary value problem (BVP).

(25 marks)

– END OF QUESTIONS –

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TABLE Q2(a)

Table 1

x_i	$f(x_i)$	$f''(x_i)$	|error|
2.0	7.3891		
2.1	8.1662		
2.2	9.0250		
2.3	9.9742		
2.4	11.023		
2.5	12.182		

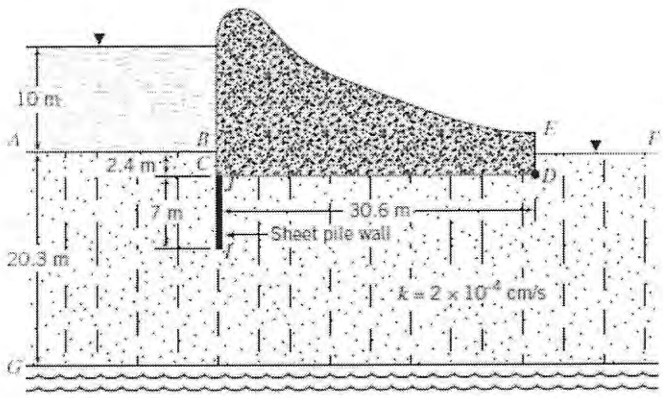

FIGURE Q2(b)

TABLE Q2(b)

i	0	1	2	3	4	5	6	7	8	9	10
Distance, x (m)	0	3.06	6.12	9.18	12.24	15.3	18.36	21.42	24.48	27.54	30.6
Head pressure, h_p	8.40	8.26	7.97	7.47	7.12	6.69	6.12	5.69	5.05	4.47	3.48

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FORMULAE

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \dots \frac{(x-x_n)}{(x_i-x_n)}; f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \dots$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)}}{a_{ii}}, \forall i - 1,2,3,\dots,n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3,\dots,n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,..,n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3,\dots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k) \quad , k = 0,1,2,3,\dots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

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Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method :$v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i.j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

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2-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

3-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) + c_3 x_3 \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$$

Lagrange Interpolating Polynomial

$$L_i(x) = \frac{(x-x_1)}{(x_i-x_1)} \cdot \frac{(x-x_2)}{(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$$

$$= \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x-x_i)}{(x_i-x_j)}$$

$$f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$$