Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ADVANCED NUMERICAL ANALYSIS
COURSE CODE	:	MWA 10103
PROGRAMME CODE	:	MWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 **Table Q1** shows the upward velocity $v(t)$ of a rocket that is given as a function of time t.

(a) Determine the value of the velocity at $t = 16\,\text{s}$ using third order Lagrange polynomial interpolation.

(8 marks)

(b) Find the absolute relative approximate error for the third order polynomial approximation in **Q1(a)** by comparing the result with that obtained using the second order Lagrange polynomial interpolation that is 392.19 m/s.

(2 marks)

(c) Evaluate the distance covered by the rocket from $t = 11\,\text{s}$ to $t = 16\,\text{s}$ using third order Lagrange polynomial interpolant for the velocity.

(10 marks)

(d) Evaluate the acceleration of the rocket at $t = 16\,\text{s}$ using the third order Lagrange polynomial interpolant for the velocity.

(5 marks)

Q2 Consider the function $f(x) = \cos(x) - x = 0$. Estimate a root of f that is accurate to the tenth decimal place using

(a) a fixed-point method with the initial value, $x = \pi/4$.

(7 marks)

(b) Newton-Raphson method with the initial value, $x = \pi/4$.

(7 marks)

(c) Secant method with the initial values, $x_0 = 0.5$ and $x_1 = \pi/4$.

(6 marks)

From the results obtained in **Q2(a)-(c)**, recommend the most suitable method by giving your justification.

(5 marks)

Q3 (a) The flow rate of an incompressible fluid in a pipe of radius 1 is given by

$$Q = \int_0^1 2\pi r V \, dr,$$

where r is the distance from centre of the pipe and V is the velocity of the fluid. Predict the value of Q if only the following tabulated velocity measurements V as shown in **Table Q3(a)** are available by using suitable Simpson's rule. Compare your result with the value obtained by solve it analytically using $V = 1 - r^2$.

(10 marks)

(b) Use Gaussian quadrature with $n = 3$ and exact arithmetic to estimate $\int_{-1}^{1} x^4 \, dx$. Compare your results to the exact value of the integral and discuss the results.

(7 marks)

(c) Evaluate $\int_{0}^{\pi/2} \cos(x) \, dx$ using the Euler-Maclaurin's formula by taking $h = \pi/4$.

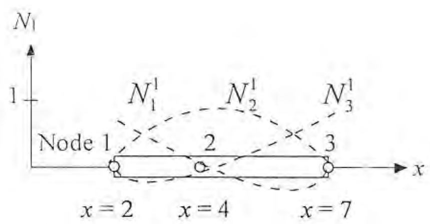
(8 marks)

Q4 Consider the heat flow equation

$$\frac{d}{dx}\left(Ak \frac{dT}{dx} \right) + Q(x) = 0, \qquad \text{for } 2 \le x \le 7,$$

on a fin consisting of three nodes and one element, as shown in **Figure Q4**. In this equation, T is the temperature at length x, A is the cross-sectional area, k is the thermal conductivity, and Q is the heat supply per unit time and per unit length. Given the following values: $A = 8$ unit, $k = 5$ unit and $Q = 60$ unit. The boundary conditions are given as $T_1 = T\,|_{x=2} = 0$ and the flux, $q_3 = q\,|_{x=7} = 40$ unit. The shape functions are defined as below:

$$N^1(x) = \begin{cases} N_1^1(x) = \dfrac{x^2}{10} - \dfrac{11}{10}x + \dfrac{14}{5}, \\ N_2^1(x) = -\dfrac{x^2}{6} + \dfrac{3}{2}x - \dfrac{7}{3}, \\ N_3^1(x) = \dfrac{x^2}{15} - \dfrac{2}{5}x + \dfrac{8}{15}. \end{cases}$$



By using Galerkin method for approximation on the quadratic model, $T = \alpha_0 + \alpha_1 x + \alpha_2 x^2$, evaluate

(a) the temperature at each nodal point, $T_2 = T\,|_{x=4}$ and $T_3 = T|_{x=7}$,

(22 marks)

(b) the flux at the left end of the fin, $q_1 = q\,|_{x=2}$.

(3 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

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Table Q1: Upward velocity $v(t)$ of a rocket

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Table Q3(a): Tabulated velocity measurements $V(r)$

r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
V	1.0	0.99	0.96	0.91	0.84	0.75	0.64	0.51	0.36	0.19	0.0

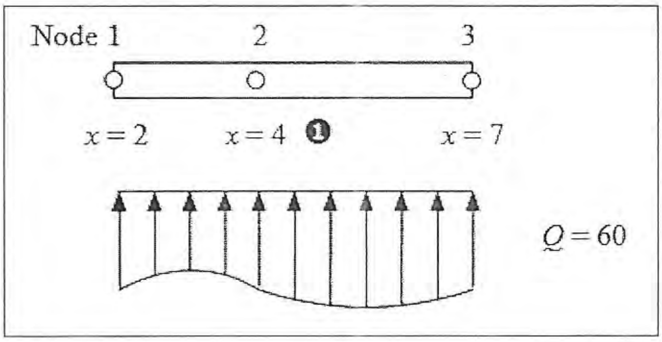

Figure Q4: Physical model of a fin