UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	DAS 10303
PROGRAMME	:	1 DAE / 3 DAE
EXAMINATION DATE	:	JUNE/JULY 2016
DURATION	:	3 HOURS
INSTRUCTIONS		

SECTION A: ANSWER **ALL** QUESTIONS.

SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY.

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

SECTION A

Q1 (a) Find the Laplace transforms for the functions below.

(i) $f(t) = \sin t$

(1 mark)

(ii) $f(t) = 4t - 3t^2 + 5e^{6t}$

(3 marks)

(iii) $f(t) = \sinh 2t + \cos 7t$

(2 marks)

(b) By using **First Shift Theorem** or **Multiply with t^n** method, find the Laplace transforms for the functions below.

(i) $f(t) = e^{-3t} \sin 2t$

(3 marks)

(ii) $f(t) = t^2 \cosh t$

(5 marks)

(c) By using the **Second Shift Theorem**, find the Laplace transforms for

$$g(t) = \begin{cases} 2 - t & 0 \le t < 7 \\ 5 & 7 \le t < 14 \\ 2t + 3 & t \ge 14 \end{cases}$$

(6 marks)

Q2 (a) Find the Inverse Laplace transform for the functions below.

(i) $F(s)=\dfrac{15}{s}-\dfrac{12}{s+3}$

(2 marks)

(ii) $F(s)=\dfrac{5}{(s-3)^2}+\dfrac{7}{(s+3)^2}$

(3 marks)

(iii) $F(s)=\dfrac{24}{s^2+1}+\dfrac{31}{s^2-9}$

(4 marks)

(b) Given $G(s)=\dfrac{100}{s^2+12-253}$.

(i) Factorise $s^2+12-253$

(2 marks)

(ii) Find the partial fractions for $G(s)$.

(5 marks)

(iii) Determine the inverse Laplace transform of $G(s)$.

(4 marks)

SECTION B

Q3 (a) Given two functions, $f: x \to x^2 + 9, x \in \Re$ and $g: x \to \dfrac{2x-3}{4}, x \in \Re$.

Calculate the value of

(i) $f(0) - g(1)$

(2 marks)

(ii) $3f(1) \cdot g(5)$

(2 marks)

(iii) x when $8g(x) + f(x) = 0$

(3 marks)

(b) Evaluate $\cos\left(\tan^{-1}\dfrac{\sqrt{x^2+1}}{2x}\right)$.

(5 marks)

(c) The function p is defined by $p: x \to 5x + 2$ The function q is such that $p \circ q: x \to \dfrac{10}{2x-3}$.

(i) Find $p(-11)$ and $p \circ q(2)$.

(2 marks)

(ii) Find $q(x)$.

(3 marks)

(iii) Find the inverse function, $(p \circ q)^{-1}$.

(3 marks)

Q4 (a) Given $\lim_{x\to 2} f(a) = 105$ and $\lim_{x\to 2} g(a) = 225$. Find

(i) $3\lim_{x\to 2} g(a) + 4\lim_{x\to 2} f(a)$

(2 markS)

(ii) $\lim_{x\to 2} \frac{f}{\sqrt{g}}(a)$

(2 marks)

(b) Calculate

(i) $\lim_{x\to 4} 7x^3 - 3x^2 + \frac{5}{2\sqrt{x}}$

(2 marks)

(ii) $\lim_{x\to 5} \frac{3x - x^2 + 10}{2\sqrt{x^2 - 4x} - \sqrt{4x}}$

(4 marks)

(c) Let $f(x) = \begin{cases} 2x - 5 & x \leq 2 \\ 3 - x^2 & 2 < x < 5 \\ 4 + \frac{24}{x} & 5 \leq x < 8 \\ 7 & x \geq 8 \end{cases}$

(i) Calculate $f(0)$, $f(3)$ and $f(10)$.

(3 marks)

(ii) Calculate $\lim_{x\to 2} f(x)$ and $\lim_{x\to 5} f(x)$.

(2 marks)

(iii) Check whether $f(0)$ is continuous at $x = 2$, $x = 5$ and $x = 8$.

(5 marks)

Q5 (a) Differentiate $y=\left(\sqrt{x^2-3x-4}\right)^3$ using extended chain rule.

(8 marks)

(b) The function $y=f(x)$ is defined by $y=\sin 3t$ and $x=2+5t^2$. Find

(i) $\frac{dy}{dx}$

(2 marks)

(ii) $\frac{d^2y}{dx^2}$

(3 marks)

(c) Use implicit differentiation to find $\frac{dy}{dx}$ if $10y^3+5x^2y-3e^y=7\sin y$.

(7 marks)

Q6 Given a function $f(x)=\frac{1}{3}x^3+\frac{7}{2}x^2-30x+5$.

(i) Find $f'(x)$, $f''(x)$, a, b, and complete the table below.

Area	$x<-10$	$-10<x<a$	$a<x<b$	$x>b$
tv	-11		0	5
f'				
f''				
Slope				
Concavity				
Shape of f				

(10 marks)

(ii) Sketch the graph of $f(x)$.

(10 marks)

Q7 (a) Find the Laplace transforms for $f(t) = 4t\sin 4t - 3e^{t}\cosh 3t$.

(5 marks)

(b) Find the inverse Laplace for $F(s) = \dfrac{2s-3}{2s^2 - 4s + 3}$.

(5 marks)

(c) Solve $3\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} = 4e^{-5t}$, given $y(0) = 10,\ y'(0) = 5$.

(10 marks)

- END OF QUESTIONS –

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FORMULAE

Differentiations

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}.\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{u}}\right) = \frac{1}{2\sqrt{u}}.\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u.\frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}.\frac{du}{dx}$$

$$\frac{d}{dx}(\mathrm{k}u) = k.\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}(\sin u) = \cos\ u.\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin\ u.\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = sec^2\, u.\frac{du}{dx}$$

$$\frac{d}{dx}(\sec\ u) = \sec\ u.tan\, u.\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -csc^2\, u.\frac{du}{dx}$$

$$\frac{d}{dx}(\csc\ u) = -\csc\ u.cot\, u.\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Parametric Differentiations

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{dy}{dt} \times \frac{dt}{dx}$$

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$$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\bigg/\frac{dx}{dt}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}$$

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FORMULAE

Laplace and Inverse Laplace Transforms

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$

The First Shift Theorem

$e^{at}f(t)$	$F(s-a)$

Multiply with t^n

$t^n f(t),\ n=1,2,..$	$(-1)^n\frac{d^nF(s)}{ds^n}$

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The Unit Step Function	
$H(t-0)$	$\frac{1}{s}$
$H(t-a)$	$\frac{e^{-as}}{s}$
The Second Shift Theorem	
$f(t-a)\,H(t-a)$	$e^{-as}\,F(s)$
***Heaviside* Function**	
$g(t)=g_1+[g_2-g_1]H(t-a)+[g_3-g_2]H(t-b)$	
Initial Value Problem	
$\mathcal{L}\{y(t)\}=Y(s)$	
$\mathcal{L}\{y'(t)\}=sY(s)-y(0)$	
$\mathcal{L}\{y''(t)\}=s^2Y(s)-sy(0)-y'(0)$	