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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2015/2016

| | | |
|---|---|---|
| COURSE NAME | : | TECHNICAL MATHEMATICS II |
| COURSE CODE | : | DAS 11103 |
| PROGRAMME | : | 3 DAJ |
| EXAMINATION DATE | : | DECEMBER 2015/JANUARY 2016 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | SECTION A) ANSWER **ALL** QUESTIONS<br><br>SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY |

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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**SECTION A**

**Q1** (a) Determine whether the following integration is improper or proper integral. Give your reason.

$$\int_{1}^{\infty} \frac{-7}{2x^2 - 2}\, dx$$

(3 marks)

(b) Evaluate $\int x^2 \ln x\, dx$ by using integration by parts.

(7 marks)

(c) Evaluate the following integral by using substitution

$$\int_{0}^{1} \frac{8x - 2}{(2x^2 - x + 3)^3}\, dx$$

(10 marks)

**Q2** (a) Solve the following integral by using Simpsons's rule, using $h = 0.125$. Write the answer to 3 decimal places

$$\int_{0}^{1} \sqrt{\frac{x}{1 + x}}\, dx$$

(9 marks)

(b) Determine the area of the region bounded by the curve and line

$$y = 2 + x - x^2\,, \; y + x + 1 = 0$$

(11 marks)

**SECTION B**

**Q3** (a) Sketch the graph and determine the domain and range.

(i) $f(x) = -x^3 - 2$

(4 marks)

(ii) $f(x) = \dfrac{1}{2x-1}$

(4 marks)

(iii) $f(x) = -\dfrac{1}{(x+1)^2}$

(4 marks)

(b) Given the function $f(x) = 3x^2 + 2$ and $g(x) = x - 2$. Find

(i) $(f^{-1} \circ g^{-1})(x)$

(5 marks)

(ii) $(g^{-1} \circ f^{-1})(5)$

(3 marks)

**Q4** (a) From **Figure Q4 (a),** evaluate the value of $\lim\limits_{x \to 1} f(x)$.

(3 marks)

(b) Compute the following limits.

(i) $\lim\limits_{x \to 4} \left(\dfrac{-2x+8}{x^2 - x - 12}\right)$

(4 marks)

(ii) $\lim\limits_{x \to -2} \left(\dfrac{x^3 + 6x^2 + 8x}{x+2}\right)$

(4 marks)

(iii) $\lim\limits_{x \to 0} \left(\dfrac{\sqrt{x+9} - 3}{x}\right)$

(5 marks)

(c) Given $h(x)=\begin{cases} x^2-1 & , x\le 1 \\ 2-k(x+1) & , x>1 \end{cases}$. Find $k$, so that $h(x)$ continuous at every value of $x$.

(4 marks)

**Q5** (a) Find the second derivative of $y = 5x^6 - 2x^5 + 3x$.

(2 marks)

(b) Find $\frac{dy}{dx}$ of the following:

(i) $y = (x^2 - 2x + 1)^{1/2}$.

(6 marks)

(ii) $4x^2 - xy + 3y = 5$

(7 marks)

(c) Given $x = t^3 - 8t$ and $y = 5 - t^4$. Calculate $\frac{dy}{dx}$ when $t = 2$.

(5 marks)

**Q6** (a) Using L'Hôspital's Rule, find

(i) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

(3 marks)

(ii) $\lim_{x\to 0} \frac{4x^3-\cos x}{x^3}$

(5 marks)

(b) The radius of a circle is increasing at the rate of $5\ m\ s^{-1}$. The formulation of area is given by $A = \pi r^2$. Find

(i) the rate of change of the area when its radius is $12\ m$.

(6 marks)

(ii) the radius of circle when its area increasing at the rate of $50\pi\ m^2 s^{-1}$.

(6 marks)

**Q7** (a) From **Figure Q7 (a)**, find the area of the region enclosed by the curve

$$y^2 = 4x \text{ and } y = 2x - 4.$$

(7 marks)

(b) Use cylindrical shells to find the volume of the solid that results when the region enclosed by $y^2 = 4x$, $y = 2$ and $x = 4$ is revolved about the $y$ – axis. Refer to **Figure Q7 (b)**.

(6 marks)

(c) Find the arc length of the curve $y = \frac{1}{3}\left(x^2 + 2\right)^{3/2}$ from $x = 0$ to $x = 3$.

(7 marks)

**- END OF QUESTION-**

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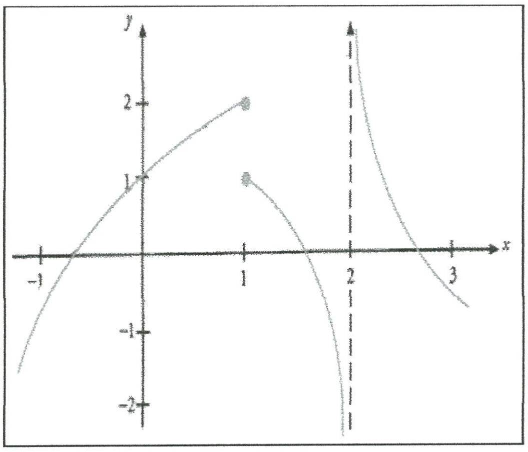


**Figure Q4 (a)**

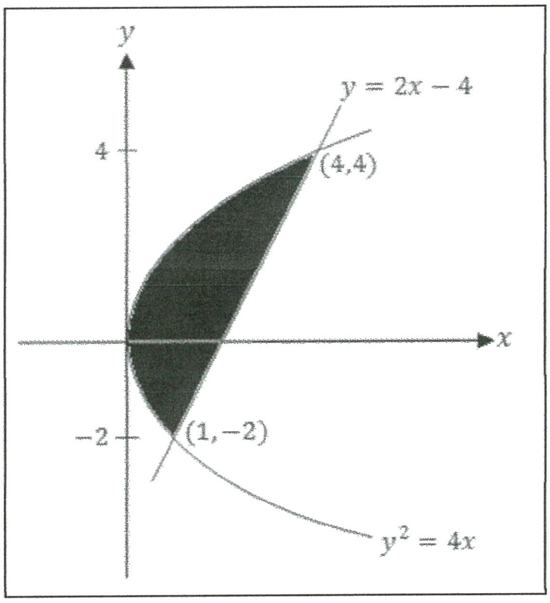


**Figure Q7 (a)**

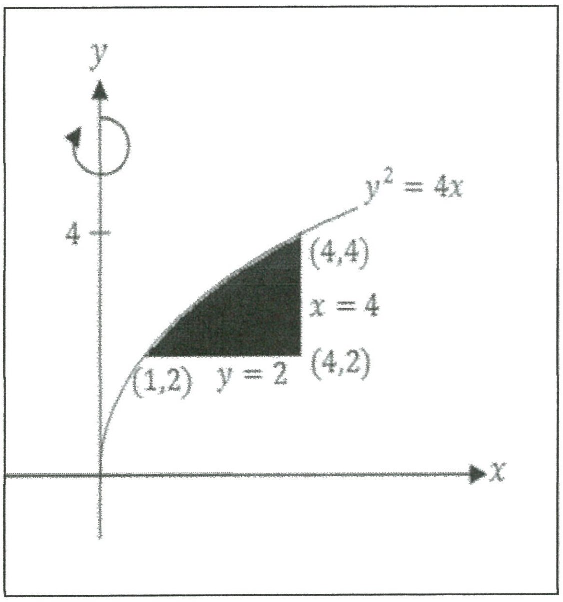


**Figure Q7 (b)**

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## FORMULAE

**Table 1: Differentiation**

| | |
|---|---|
| $\frac{d}{dx}(ax^n) = nax^{n-1}$ | $\frac{dy}{dx} = \frac{dy}{dm} \cdot \frac{dm}{dn} \cdot \frac{dn}{dx}$ |
| $\frac{d}{dx}(e^x) = e^x$ | $\frac{d}{dx}(\sin x) = \cos x$ |
| $\frac{d}{dx}(a^x) = a^x \ln a$ | $\frac{d}{dx}(\cos x) = -\sin x$ |
| $\frac{d}{dx}(\ln x) = \frac{1}{x}$ | $\frac{d}{dx}(\tan x) = \sec^2 x$ |
| $\frac{d}{dx}(\log_b x) = \frac{1}{x}\log_b e$ | $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cdot \cot x$ |
| $\frac{d}{dx}(uv) = uv' + vu'$ | $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$ |
| $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$ | $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$ |
| $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ | |

**Table 2: Integration**

| | |
|---|---|
| $\int k\,dx = kx + C$ | $\int e^x\,dx = e^x + C$ |
| $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ | $\int \sin x\,dx = -\cos x + C$ |
| $\int \frac{1}{x}\,dx = \ln x + C$ | $\int \cos x\,dx = \sin x + C$ |
| **Definite Integral**<br>$\int_a^b f(x)\,dx = F(b) - F(a)$ | **Integration by Parts**<br>$\int u\,dv = uv - \int v\,du$ |

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**Table 2: Integration**

**Trapezoidal Rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{i=1}^{n-1} f(a+ih)\right]$$

**Simpson Rule**

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+f(b)+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f(a+ih)+2\sum_{\substack{i=1\\ i\text{ even}}}^{n-1} f(a+ih)\right]$$

**Area**

$$A=\int_a^b \left[f(x)-g(x)\right]dx$$

$$A=\int_c^d \left[f(y)-g(y)\right]dy$$

**Volume in Cylindrical Shells**

$$V=2\pi\int_a^b x\left[f(x)-g(x)\right]dx$$

$$V=2\pi\int_c^d y\left[f(y)-g(y)\right]dy$$

**Arc Length**

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$