CONFIDENTIAL

UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	TECHNICAL MATHEMATICS I
COURSE CODE	:	DAS 11003
PROGRAMME	:	1 DAB/ 1 DAJ/ 1 DAR/ 1 DAK
EXAMINATION DATE	:	DECEMBER 2015/ JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A) ANSWER **ALL** QUESTION. SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY.

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

CONFIDENTIAL

CONFIDENTIAL

SECTION A

Q1 (a) Solve the following matrix operations

(i)

$$3\begin{pmatrix} 2 & 5 & -6 \\ 0 & 11 & 10 \\ -5 & 4 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -5 & 20 \\ 12 & -22 & 33 \\ 15 & 12 & 11 \end{pmatrix}$$

(3 marks)

(ii)

$$\begin{pmatrix} 7 & 8 & 9 \\ 10 & -4 & 6 \\ 3 & -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 5 & 10 \\ -6 & 13 \end{pmatrix}$$

(3 marks)

(iii)

$$\frac{2}{3}\begin{pmatrix} 9 & 0 & 5 & 3 \\ 6 & 7 & 3 & 8 \\ 1 & 2 & 0 & 8 \end{pmatrix}^T + \begin{pmatrix} 1 & 2 \\ 1 & 8 \\ 1 & 0 \\ 2 & 9 \end{pmatrix}\begin{pmatrix} 7 & 1 & 5 \\ 9 & 4 & 5 \end{pmatrix}$$

(4 marks)

(b) (i) Given $Q = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & 1 \end{pmatrix}$. Find the inverse matrix of Q, Q^{-1}.

(7 marks)

(ii) If $Q\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 6 \end{pmatrix}$, calculate x, y and z by using inversion method.

(3 marks)

CONFIDENTIAL

Q2 (a) If $A = \begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 4 & 3 \\ 10 & -7 & -6 \\ -8 & 6 & 5 \end{pmatrix}$. Calculate

(i) $4A - B$

(3 marks)

(ii) AB

(3 marks)

(b) Given the matrix equation $2A - 3\begin{pmatrix} 1 & 2 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}$. Find the matrix A.

(4 marks)

(c) Given a system of linear equation

$$x + 2y + z = 4$$
$$3x - 4y - 2z = 2$$
$$5x + 3y + 5z = -1$$

(i) Write the linear equation system into augmented matrix, $[A|B]$.

(1 mark)

(ii) Apply row-operation as given below to the augmented matrix.
Row operation :

$$R_2 - 3R_1 \rightarrow R_2$$
$$R_3 - 5R_1 \rightarrow R_3$$
$$R_2 \leftrightarrow R_3$$
$$\frac{R_2}{-7} \rightarrow R_2$$

(4 marks)

(iii) Find x, y and z by continuing the **Q6 (b) (ii)** by using Gauss-Jordan elimination method.

(5 marks)

SECTION B

Q3 (a) Calculate the value of x and y. Given

(i) $3x + y = 7$ and $4x - 3y = 18$

(3 marks)

(ii) $16^{3x} \cdot 4^{2y} = 16$ and $9^{2x} \cdot 27^{y} = 9$

(4 marks)

(b) (i) Simplify and rationalize the denominator $\dfrac{\sqrt{11} - \sqrt{5}}{3\sqrt{5} + 2\sqrt{11}}$.

(3 marks)

(i) Given $x = \dfrac{5}{3 - \sqrt{6}}$ and $y = \dfrac{4 - \sqrt{7}}{2}$. Calculate and simplify $12xy - 11x$.

(4 marks)

(c) (i) Without using calculator, simplify $\log_3\left(\dfrac{81 \times 27^{x}}{2187^{y}}\right)$.

(3 marks)

(ii) Given $\log_5 x = 3$ and $\log_y 32 = 5$, calculate $4 + 3x - 7y$.

(3 marks)

Q4 (a) Given that $A = 3x + 4$, $B = 5 - 2x$ and $C = x^2$. Evaluate

(i) $AB + 7C$

(3 marks)

(ii) $B^2 - A$

(3 marks)

(b) Solve the inequality $3x^2 \le 2 - x$.

(5 marks)

(c) Express $\dfrac{6x^2 - 9x - 18}{x^2(x+6)}$ in partial fraction.

(5 marks)

(d) Find the root of x of the equation $f(x) = x^3 - 5x^2 - 4x + 3$ in between [5 , 6] by using bisection method. Iterate until $|f(x_i)| \le 0.005$.

(4 marks)

Q5 (a) Give that the n^{th} term of a sequence is $a_n = 2(n-1)^2 - 3$. Write the first five terms of the sequence.

(7 marks)

(b) A ball dropped from a roof of a house and bounced back. The first bounced height is 8m. The ball bounce back at 80% of its previous bounce height until it is completely stop.

(i) Determine the type of progression of height of the ball bounce, whether it is Arithmetic progression or Geometric progression.

(5 marks)

(ii) Find the height of the fifth bounce.

(2 marks)

(c) Expand $(x+3y)^5$

(3 marks)

(d) Find the 6^{th} term of $(2x-5y)^7$

(3 marks)

Q6 (a) Prove that $\sec^2\theta \cdot \cos^2\theta = \sin^2\theta + \cos^2\theta$.

(3 marks)

(b) Without using calculator, find the exact value of

(i) $\cos\dfrac{2\pi}{3}$

(3 marks)

(ii) $\tan\dfrac{7\pi}{12}$.

$\left[\text{Hint}: \dfrac{7\pi}{12} = \dfrac{\pi}{3} + \dfrac{\pi}{4}\right]$

(3 marks)

(c) Solve for θ for the expression given within the interval $[0, 2\pi]$.

(i) $\dfrac{3}{2}\sin\theta = -\dfrac{1}{2}$

(4 marks)

(ii) $\tan^2\theta + \tan\theta - 2 = 0$

(7 marks)

Q7 (a) Given $A=\begin{pmatrix}4 & 2\\ 3 & -5\\ -1 & 6\end{pmatrix}$, $B=\begin{pmatrix}1 & 2 & 4\\ 2 & 1 & -5\end{pmatrix}$ and $C=\begin{pmatrix}-2 & 3\\ 4 & -5\end{pmatrix}$.

Calculate

(i) $5B-A^T$

(3 marks)

(ii) AB

(3 marks)

(iii) BB^T+C

(5 marks)

(b) Given a system of linear equation

$$-2x+6y+z=9$$
$$-x+y-7z=-6$$
$$4x-y-z=3$$

(i) Rearrange the system of linear equation given into diagonally dominant matrix.

(1 mark)

(ii) Rewrite the equation of x, y and z from the diagonally dominant system of linear equation.

(3 marks)

(iii) Find x, y and z by using Gauss-Seidel Iteration mathod with initial value (0, 0, 0). Iterate until maximum error is less than 0.001.

(5 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER/SESSION : SEM I/2015/2016 PROGRAMME : 1 DAB/DAJ/DAR/DAK
COURSE NAME : TECHNICAL MATHEMATICS I COURSE CODE : DAS11003

Exponent, Radical & Logarithms

i) $x^m . x^n = x^{m+n}$

ii) $\frac{x^m}{x^n} = x^{m-n}$

iii) $(x^m)^n = x^{mn}$

iv) $x^{p/q} = \left(\sqrt[q]{x}\right)^p$

v) $x = b^n \Leftrightarrow \log_b x = n$

vi) $\log_b(xy) = \log_b x + \log_b y$

vii) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

viii) $\log_b x^k = k \log_b x$

ix) $\log_a x = \frac{\log_b x}{\log_b a}$

Polynomial

i) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ii)
$$x^2 + bx + c = x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$
$$= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

iii) $x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

Sequence & Series

i) $\sum_{k=1}^{n} c = cn$

ii) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

iii)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Arithmetic Series

i) $T_n = a + (n-1)d$

$d = u_n - u_{n-1}$

ii) $S_n = \frac{n}{2}(a + u_n)$

iii)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series

i) $T_n = ar^{n-1}$

ii) $r = \frac{u_n}{u_{n-1}}$

iii) $S_n = \frac{a(1-r^n)}{1-r}$

iv) $S_\infty = \frac{a}{1-r}$

FINAL EXAMINATION

SEMESTER/SESSION : SEM I/2015/2016 PROGRAMME : 1 DAB/DAJ/DAR/DAK
COURSE NAME : TECHNICAL MATHEMATICS I COURSE CODE : DAS11003

The Binomial Theorem

i) $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

ii) $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + \binom{n}{n}b^n$

iii) $(r+1)th = \binom{n}{r}a^{n-r}b^r$

Trigonometric Identity

i) $\cos^2\theta + \sin^2\theta = 1$

ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $\cot^2\theta + 1 = \csc^2\theta$

Addition and Subtraction Formulas:

i) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

ii) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

iii) $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Double - Angle Formulas

i) $\sin 2\theta = 2\sin\theta\cos\theta$

ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
OR $\cos 2\theta = 2\cos^2\theta - 1$
OR $\cos 2\theta = 1 - 2\sin^2\theta$

iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half – Angle Formulas

i) $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

ii) $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

iii) $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

CONFIDENTIAL

FINAL EXAMINATION

SEMESTER/SESSION : SEM I/2015/2016 PROGRAMME : 1 DAB/DAJ/DAR/DAK
COURSE NAME : TECHNICAL MATHEMATICS I COURSE CODE : DAS11003

Trigonometry Equation in the Form: $a \sin \theta + b \cos \theta = c$

Let
$$\begin{aligned} a\sin\theta + b\cos\theta &= r\sin(\theta+\alpha) \\ &= r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) \\ &= (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta \end{aligned}$$

We get $a = r\cos\alpha$ and $b = r\sin\alpha$ $\Rightarrow r = \sqrt{a^2+b^2}$ $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

We use the above to solve:

$$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha)$$
$$a\sin\theta - b\cos\theta = r\sin(\theta-\alpha)$$
$$a\cos\theta + b\sin\theta = r\cos(\theta-\alpha)$$
$$a\cos\theta - b\sin\theta = r\cos(\theta+\alpha)$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = (c_{ij})^T$$

$$A^{-1} = \frac{1}{|A|} Adj(A)$$