UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20603
PROGRAMME	:	2 DAE
EXAMINATION DATE	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A) ANSWER **ALL** QUESTIONS SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

SECTION A

Q1 (a) Solve the initial value problem of the second order homogeneous differential equation below.

$$y'' + 25y = 0, \qquad y(0) = 3, \qquad y'(\pi) = 2$$

(7 marks)

(b) Using method of variation of parameter, find the general solution of the following second order non-homogeneous differential equation below.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 5e^{2x}$$

(13 marks)

Q2 Using method of undetermined coefficients, find the general solution of the given second order differential equations.

(a) $y'' + 9y = 4x^2 + 2e^{3x}$

(12 marks)

(b) $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 16y = 3e^{3x}$

(8 marks)

SECTION B

Q3 (a) By using substitution technique, evaluate

$$\int_{1}^{2}\left(\frac{2x}{(x^2+3)^3}\right)dx+\int_{3}^{4}5x\,dx$$

(7 marks)

(b) Evaluate by using tabular method

$$\int \frac{3}{2}x^3e^{2x}\,dx$$

(7 marks)

(c) Solve the following integral by using Trapezoidal's rule, using $h=0.125$.
Write the answer to 3 decimal places

$$\int_{1}^{2}\frac{\sqrt[3]{x^2+7x}}{e^{2x}}\,dx$$

(6 marks)

Q4 (a) Show that the area of the region bounded by the curve $y=4-x^2$ and the line $y=4-2x$ is $1\frac{1}{3}$ unit^2.

(9 marks)

(b) **Figure Q4(b)** shows the curve $y=\frac{2}{9}x^2$ and the line $y=5-x$. Determine

(i) coordinates of A and B.

(3 marks)

(ii) volume of the solid generated when the bounded region revolves 360° about $y-$axis using cylindrical shells.

(8 marks)

Q5 Given a first order differential equation

$$(3+\ln x-\frac{y}{x})dx+(2-\ln x)dy=0.$$

(a) Show that the equation is exact equation.

(5 marks)

(b) Find the general solution to the differential equation.

(12 marks)

(c) Find the particular solution of the equation if $y(4)=2$.

(3 marks)

Q6 The temperature of a dead old man when it was found at 5:00 am is $34°C$. The surrounding temperature has been kept at a constant $29°C$. After two hours, the temperature of the body is taken once more and found to be $30°C$. Assuming that the victim's body temperature was normal $(36°C)$ prior to death.

(a) By following the Newton's Law of Cooling,

$$\frac{dT}{dt}=-k(T-T_s).$$

Find the cooling equation of the body if the initial condition $T=T_0$.

(10 marks)

(b) Determine the time of the death.

(10 marks)

Q7 The investigation on the human skull discover that it contains 25% of carbon named C-14. It is known that the material exponential decay can be formulated as

$$\frac{dN}{dt} = kN$$

where N is the amount of material present and k is constant of proportionally.

(a) By using separable differential equation, solve the population decay.

(4 marks)

(b) Assuming that the half life of carbon is 2500 years, interpret the value of k and find how old is the skull if it is found now.

(7 marks)

(c) Find the duration needs to get 0.1% carbon in that skull.

(4 marks)

(d) Find the remaining percentage of carbon after 40,000 years after discovery.

(5 marks)

- END OF QUESTION –

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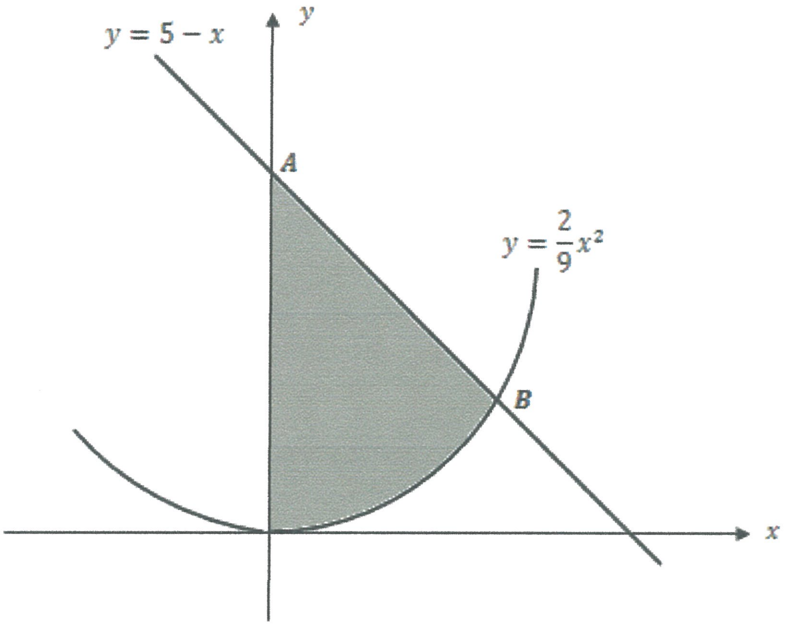

Figure Q4 (b)

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Formulae

Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Method of Variation of Parameters

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW}\,dx$ $u_2 = \int \frac{y_1 f(x)}{aW}\,dx$

Particular solution, $y_p = u_1 y_1 + u_2 y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$

Method Of Undetermined Coefficients

Case	$F(x)$	$y_p(x)$
1	Simple polynomial: $A_0 + A_1 x + \ldots + A_n x^n$	$x^r(B_0 + B_1 x + \ldots + B_n x^n)$, $r = 0, 1, 2,$
2	Exponential function: $Ce^{\alpha x}$	$x^r(ke^{\alpha x})$, $r = 0, 1, 2,$
3	Simple trigonometry: $C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$, $r = 0, 1, 2,$

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Trigonometry

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$2\sin x \cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x \cos y = \cos(x+y) + \cos(x-y)$$

Differentiation and Integration

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x \rvert + C$
$\frac{d}{dx}\ln\lvert ax+b\rvert = \frac{1}{ax+b}$	$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin ax = a\cos ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\frac{d}{dx}\cos ax = -a\sin ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$

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Definite Integration

$$\int_a^b f(x)\,dx \quad = F(b) - F(a)$$

Length of Curve

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Area

$$A = \int_a^b [f(x) - g(x)]\,dx$$
$$= \int_c^d [f(y) - g(y)]\,dy$$

Volume Cylindrical Method

$$V = 2\pi\int_a^b x\,f(x)\,dx$$
$$= 2\pi\int_c^d y\,f(y)\,dy$$

Area of Surface Revolution

$$S = 2\pi\int_a^b y(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= 2\pi\int_c^d x(y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$