Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2 DAA / 2 DAM / 3 DAA
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **PART A** B) ANSWER **THREE (3)** QUESTIONS ONLY IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A

Q1 (a) Find the inverse of the following Laplace transform.

(i) $\dfrac{6s+3}{s^2+25}$

(4 marks)

(ii) $\dfrac{1}{s^4}+\dfrac{1}{2s+8}-\dfrac{4}{s-3}$

(5 marks)

(iii) $\dfrac{8}{3s^2+12}-\dfrac{3}{s^2-49}$

(5 marks)

(b) (i) Express $\dfrac{s+7}{s^2-3s-10}$ as partial fraction.

(3 marks)

(ii) Find the inverse Laplace of the partial fraction from **Q1(b)(i)**.

(3 marks)

Q2 Solve the following differential equation by using Laplace transform.

(a) $y'+4y=e^{-4t}, \quad y(0)=2$

(8 marks)

(b) $y''-6y'+8y=0, \quad y(0)=0, \quad y'(0)=-3$

(12 marks)

PART B

Q3 (a) Given $(3x^2 + \frac{7}{3}xy^3)\,dx + (\frac{7}{2}x^2y^2 - 2y^2)\,dy = 0$.

(i) Show that the differential equation above is an exact equation.

(3 marks)

(ii) Then, solve the equation.

(8 marks)

(b) Given ordinary linear differential equation $x^2\frac{dy}{dx} + 2xy = xe^x$.

(i) Find $p(x)$ and $q(x)$.

(3 marks)

(ii) Thus, solve the equation.

(6 marks)

Q4 (a) The rate of cooling of a body is given by the equation

$$\frac{dT}{dt} = -k(T - 10)$$

where T is the temperature in degrees Celsius, k is a constant and t is the time in minutes. When $t = 0$, $T = 90^\circ C$ and when $t = 5$, $T = 60^\circ C$. Show that when $t = 10$, $T = 41.25^\circ C$.

(10 marks)

(b) A group of virus is grown under ideal conditions in a laboratory, the virus population increases at a rate proportional to the amount present. At the end of 5 hours, there are 20,000 virus and at the end of 8 hours, there are 50,000 virus. Find the amount of virus initially present.

(10 marks)

Q5 (a) Given a nonhomogeneous differential equations of $y'' - 4y' - 12y = 12x^2 - 5 + 3e^x$. Compute

(i) the homogeneous solution of the equation.

(3 marks)

(ii) the particular solution of the nonhomogeneous differential equation of $y'' - 4y' - 12y = 12x^2 - 5$.

(8 marks)

(iii) the particular solution of the nonhomogeneous differential equation of $y'' - 4y' - 12y = 3e^x$.

(6 marks)

(b) From the answer in **Q(a)(i), Q(a)(ii)** and **Q(a)(iii)**, write the general solution of the nonhomogeneous equation.

(3 marks)

Q6 Find the Laplace transform of following functions.

(a) $f(t) = 3 + 12t^2$

(3 marks)

(b) $f(t) = \sinh 7t - e^{-\frac{1}{2}t}$

(4 marks)

(c) $f(t) = (t+1)^3$

(5 marks)

(d) $f(t) = t^2 \sin 2t$

(8 marks)

Q7 (a) Solve the second order homogeneous differential equation of $y'' - 3y' + 2y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$.

(7 marks)

(b) Show that

$$\mathcal{L}\left\{\cos^2 2t - \sin^2 2t\right\} = \frac{s}{s^2 + 16}$$

(3 marks)

(c) Use Laplace Transform to solve the differential equation $y' + y = \sin 2t$, given $y(0) = 0$.

(10 marks)

– END OF QUESTION –

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Formulae

Table 1 : Laplace Transformation

$f(t)$	$F(s)$	$f(t)$	$F(s)$
k	$\frac{k}{s}$	$\sinh at$	$\frac{a}{s^2-a^2}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$	$\cosh at$	$\frac{s}{s^2-a^2}$
e^{at}	$\frac{1}{s-a}$	$e^{at}f(t)$	$F(s-a)$
$\sin at$	$\frac{a}{s^2+a^2}$	$t^n f(t),\ n=1,2,..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\cos at$	$\frac{s}{s^2+a^2}$		

$$L\{y(t)\} = Y(s)$$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

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Table 2 : Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{ds}(uv) = v\frac{du}{ds} + u\frac{dv}{ds}$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b}\ln\lvert a-bx\rvert + C$	$\frac{d}{ds}\left(e^{ax}\right) = ae^{ax}$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$	$\frac{d}{ds}(\sin ax) = a\cos ax$
$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$	$\frac{d}{ds}(\cos ax) = -a\sin ax$
$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$	$\frac{d}{ds}\left(x^n\right) = nx^{n-1}$
$\int u dv\, dx = uv - \int v du$	$\frac{d}{ds}(uv) = v\frac{du}{ds} + u\frac{dv}{ds}$

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Table 3 : Characteristic Equation and General Solution

Homogeneous Differential equation : $ay'' + by' + cy = 0$

Characteristic equation : $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 4 : Particular Solution of Nonhomogeneous Equation

$$ay'' + by' + cy = f(x)$$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C\cos\beta x$ atau $C\sin\beta x$	$x^r (p \operatorname{kos} \beta x + q \sin \beta x)$
$P_n(x)e^{\alpha x}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}$
$P_n(x)\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, sin\, \beta x$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_p(x)$ and $y_h(x)$.

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Table 5 : Variation of Parameters Method.

Homogeneous solution, $y_h(x) = Ay_1 + By_2$	
Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$	
$u_1 = -\int \frac{y_2 f(x)}{aW} dx + A$	$u_2 = \int \frac{y_1 f(x)}{aW} dx + B$
General solution, $y(x) = u_1y_1 + u_2y_2$	

Table 6 : Trigonometry Identities

$\sin^2 t + \cos^2 t = 1$
$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$
$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$

Table 7 : Partial Fraction

$\frac{a}{(s+b)(s-c)} = \frac{A}{s+b} + \frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$
$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$