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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	DAS 10303
PROGRAMME	:	3 DAE
EXAMINATION DATE	:	DECEMBER 2015/JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN SECTION A. B) ANSWER **THREE (3)** QUESTIONS ONLY IN SECTION B.

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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SECTION A

Q1 (a) Find the Laplace Transforms for the functions below.

(i) $f(t) = \cos t$

(2 marks)

(ii) $f(t) = 3t^2 - 5t + e^{-t}$

(3 marks)

(iii) $f(t) = e^t \sin t$

(4 marks)

(b) Given $g(t) = \begin{cases} 3, & 0 \le t < 7, \\ t-4, & 7 \le t < 10, \\ t+7, & t \ge 10. \end{cases}$

(i) Sketch the graph for the function $g(t)$.

(3 marks)

(ii) Write the Heaviside Function of $g(t)$.

(3 marks)

(iii) By using the Second Shift Theorem, find the Laplace Transforms for $g(t)$.

(5 marks)

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Q2 (a) Find the inverse Laplace transforms for the functions below.

(i) $F(s) = \dfrac{10}{s}$

(2 marks)

(ii) $F(s) = \dfrac{3}{(s-4)^2} + \dfrac{7}{s+4}$

(3 marks)

(iii) $F(s) = \dfrac{6s}{(s-1)^2} - \dfrac{5}{s^2-4}$

(5 marks)

(b) Given $G(s) = \dfrac{2s-3}{s^2+3s-10}$

(i) Factorise $s^2 + 3s - 10$

(2 marks)

(ii) Find the partial fraction for $G(s)$.

(6 marks)

(iii) Determine the inverse Laplace of $G(s)$.

(2 marks)

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SECTION B

Q3 (a) Given $f(x) = 5x^2 - kx + 7$. Find the value of k if

(i) Find $f(0)$.

(1 mark)

(ii) Find the value of k if $f(2) = 51$.

(3 marks)

(iii) Find the value of k if $f(-1) = k(k - 3)$.

(3 marks)

(b) Given $g(x) = (x - 4)^2 - 5$.

(i) Find $g(0)$ and $g(5)$

(2 marks)

(ii) Sketch the graph of $g(x)$ for $0 \leq x < 5$.

(2 marks)

(c) The function h is defined by $h : x \rightarrow 2x - 3$. The function k is such that

$$h \circ k : x \rightarrow \frac{1}{x^2 - 2}$$

(i) Find $h(1)$ and $h \circ k(1)$.

(2 marks)

(ii) Find the inverse function of h, $h^{-1}(x)$.

(3 marks)

(iii) Find the function k.

(4 marks)

Q4 (a) Let $f(x) = \begin{cases} -4 & x < 0 \\ 4x+5 & 0 \le x \le 5 \\ x^2 & x > 5 \end{cases}$. Find

(i) $f(0)$ and $f(5)$

(2 marks)

(ii) $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$

(2 marks)

(iii) $\lim_{x\to 0} f(x)$ and $\lim_{x\to 5} f(x)$

(4 marks)

(b) Referring to **Q4(a)**, check whether

(i) $f(0) = \lim_{x\to 0} f(x)$

(2 marks)

(ii) $f(x)$ continues at $x = 0$ and $x = 5$

(3 marks)

(c) Given $\lim_{x\to 1} g(x) = 2$ and $\lim_{x\to 1} h(x) = 10$. Calculate

(i) $5 \lim_{x\to 1} g(x) - 4 \lim_{x\to 1} h(x)$

(2 marks)

(ii) $\lim_{x\to 1} \sqrt{g^2(x) + h^2(x)}$

(3 marks)

(iii) $\lim_{x\to 1} \left(\dfrac{2g(x) - h(x)}{g(x).h(x)} \right)^2$

(2 marks)

Q5 (a) Differentiate:

(i) $y = 4x + 3\sin x$

(2 marks)

(ii) $y = 2x^{10} - 5\cos(x + 1)$

(2 marks)

(iii) $y = \frac{2e^{3x}}{x\,sin\,(x+1)}$

(3 marks)

(b) Given $y = 5t + 4e^{2-t}$ and $x = 2t^2 - \tan t$.

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

(2 marks)

(ii) Find $\frac{dy}{dx}$ by using the parametric differentiations

(2 marks)

(c) (i) Find $\frac{d}{dx}(x\sin x)$

(2 marks)

(ii) Find $\frac{d}{dx}(2xy)$

(2marks)

(iii) Find the implicit differentiation for $3y^2 - 2x\sin x = 2xy$

(5 marks)

Q6 (a) A tennis ball flew horizontally after being hit by Roger at *t* second, with a formula, $s = t^3 - 2t^2 + t + 5$ meter.

Calculate

(i) the ball's velocitiy and acceleration at time t.

(2 marks)

(ii) the ball's acceleration when the velocity is zero.

(4 marks)

(iii) The ball's speed when the acceleration is 14 ms^{-2}.

(3 marks)

(b) Given a function, $f(x) = x^3 - 3x^2 - 24x + 7$.

(i) Write the first and second derivatives of *f*(*x*) and find all the minimum, maximum and inflection points.

(5 marks)

(ii) By using a table, complete it with all the important values such as $f'(x)$, $f''(x)$ and so on.

(5 marks)

(iii) Sketch the graph of *f*(*x*).

(1 mark)

Q7 (a) Find the Laplace transforms for the functions below.

(i) $f(t) = \sinh 6t$

(2 marks)

(ii) $f(t) = \sinh 2t + \cos 5t$

(2 marks)

(iii) $f(t) = te^{2t} - 3(t+5)^2$

(3 marks)

(b) Find the inverse Laplace transforms for the functions below.

(i) $F(s) = \frac{5}{s^2-16} + \frac{25}{s-10}$

(2 marks)

(ii) $F(s) = \frac{3}{4(s-11)} - \frac{4s}{s^2-144}$

(2 marks)

(c) (i) Find A and B from the equation:

$$\frac{10x+1}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

(3 marks)

(ii) Solve $\frac{dy}{dt} - 3y = 41e^{4i}$, given $y(0) = 10$.

(6 marks)

- END OF QUESTIONS -

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FORMULAE

Differentiations

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

$$\frac{d}{dx}(\sin u) = \cos\ u.\frac{du}{dx}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}.\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin\ u.\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{u}}\right) = \frac{1}{2\sqrt{u}}.\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = sec^2\,u.\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u.\frac{du}{dx}$$

$$\frac{d}{dx}(\sec\ u) = \sec\ u.tan\,u.\frac{du}{dx}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}.\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -csc^2\,u.\frac{du}{dx}$$

$$\frac{d}{dx}(\mathrm{k}u) = k.\frac{du}{dx}$$

$$\frac{d}{dx}(\csc\ u) = -\csc\ u.cot\,u.\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

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Laplace and Inverse Laplace Transforms

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
The First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t),\ n=1,2,..$	$(-1)^n \frac{d^nF(s)}{ds^n}$
The Unit Step Function	
$H(t-0)$	$\frac{1}{s}$
$H(t-a)$	$\frac{e^{-as}}{s}$
The Second Shift Theorem	
$f(t-a)\,H(t-a)$	$e^{-as}F(s)$
***Heaviside* Function**	
$g(t)=g_1+[g_2-g_1]H(t-a)+[g_3-g_2]H(t-b)$	