UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	MATHEMATICS ENGINEERING IV
CODE	:	BSM 3913
COURSE	:	2 BFA / BFB / BFF / BFP / BEE / BEM / BET 3 BEE / BDD 4 BEE
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B.** ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES.

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) A string is tightly stretched between $x = 0$ and $x = L$ and is initially at rest. Each point of the string is given an initial velocity of

$$y_t(x,0) = \mu \sin^3\left(\frac{\pi x}{L}\right).$$

Find numerically the displacement of the string with time $t = 0\ (0.5)\ 1.0$, assuming $y_{tt} = \alpha^2 y_{xx}$, $0 \le x \le L$ by taking $\alpha = 1$, $\mu = 1$, $\Delta x = 0.5$ and $L = 3.0$.

(9 marks)

(b) Given the Poisson's equation

$$u_{xx} + u_{yy} = 8x^2 y^2,$$

with boundary conditions $u(x,0) = u(x,1) = u(0,y) = u(1,y) = 0$ for $0 < x < 1$ and $0 < y < 1$. By taking $\Delta x = \Delta y = 1/3$, use finite-difference method to derive a system of linear equations that approximate the solution for the square region. (Do **NOT** solve the system)

(11 marks)

Q2 Consider the heat flow equation

$$\frac{d}{dx}\left(A(x)\, k(x)\, \frac{dT}{dx} \right) + Q(x) = 0, \text{ for } 2 \le x \le 8$$

on a fin consisting of four nodes and three elements. In this equation, $T(x)$ is the temperature at length x, $A(x)$ is the cross-sectional area, $k(x)$ is the thermal conductivity and $Q(x)$ is the heat supply per unit time and per unit length.

Given that $A(x) = 20$ unit, $k(x) = 4$ unit and $Q(x) = 50$ unit. The boundary conditions are given as $T_1 = T\big|_{x=2} = 0$ and $T_4 = T\big|_{x=8} = 0$.

Find the temperature at each nodal point, $T_2 = T\big|_{x=4}$ and $T_3 = T\big|_{x=6}$ by using finite-element method with considering only the first element and assembly technique.

(20 marks)

PART B

Q3 (a) A man was found dead from a stabbed wound in his house early in the morning. The police who came to crime scene recorded the body temperature of the decease at 27^0C. The temperature of the house is assumed to be uniform at 24^0C. Given the mathematical model of the crime as:

$$\theta(t) = \theta(0)e^{-kt} + \theta_r(1 - e^{-kt})$$

where :

$\theta(t)$ - the body temperature at time, t hours.

θ_r - the temperature in the house .

Let $\theta(0) = 37^0 C$ and $k = 0.154$, estimate how long he has been killed by using

(i) Newton – Raphson Method. Begin the calculation with $t_0 = 0$.

(ii) Secant Method for the intervals of $[0, 10]$.

(For both methods iterate until $|f(t_i)| < \varepsilon = 0.005$)

Then, if the true time is $t^* = 9.522$ hours. Find the absolute errors for both methods.

(13 marks)

(b) Given

$$\begin{aligned} 2x_1 + 5x_2 + 2x_3 &= 8 \\ 5x_1 + 2x_2 \quad\quad &= -2 \\ 2x_2 + 5x_3 &= 3 \end{aligned}$$

By taking initial guess as $x^{(0)} = (-1.220 \quad 2.176 \quad -0.270)^T$, solve it by using Gauss-Seidel iteration method and iterate until $\max\left\{ | x_i^{(k+1)} - x_i^{(k)} | \right\} < \varepsilon = 0.005$.

(7 marks)

Q4 (a) A car traveling along a rural highway has been clocked at a number of points. The data from the observations are given in the Table 1, where the time is in seconds, s and the distance is in metre, m.

Observation of a car traveling along a rural highway

Time, t	0	3	5	8	13
Distance, d	0	70	116	190	303

Table 1

Use Newton divided difference method to predict the position of the car when $t = 10\,s$.

(6 marks)

(b) Construct the natural cubic spline for the points (4,2), (9,3) and (16,4). Hence, find the approximation of $f(7)$ and $f(14)$.

(10 marks)

(c) Given $f(x)=\sqrt{\cot x}$. Find the approximate value(s) of $f'(0.05)$ with $h=0.01$ by using

(i) 2 – point backward difference formula,
(ii) 3 – point central difference formula,
(iii) 3 – point forward difference formula,
(iv) 5 – point difference formula.

Then, find the relative error for each answer if the exact answer is –44.777.

(4 marks)

Q5 (a) Suppose that the age in days of a type of single-celled organism can be expressed as $f(x)=(\ln 2)e^{-xk}$ where $k=\frac{1}{2}\ln 2$ and the domain is $0\le x\le 2$. Given that mean $=\mu=\int_0^2 f(x)\,dx$, find the mean age of the cells by using

(i) 1/3 Simpson method with $h=0.2$.

(ii) 2-point Gauss quadrature.

(10 marks)

(b) Solve $y'y^2=x^2+7x+3$ at $x=0(0.2)1$ by Euler's method with initial condition $y(0)=3$.

(4 marks)

(c) Given the boundary value problem $x''+4x=\sin t$, $0\le t\le 1$, with condition $x(0)=0$ and $x(1)=0$. Derive the system of linear equations (in matrix-vector form) using finite difference method by taking $\Delta t=h=0.25$.

(6 marks

Q6 (a) Given that the dominant eigenvalue, $\lambda_{l\arg est}$, is 13.262, find the smallest (in absolute) eigenvalue for matrix A below using shifted power method.

$$A=\begin{pmatrix} 8 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 13 \end{pmatrix}. \quad \text{Use } v^{(0)}=(1\quad 1\quad 1)^T.$$

(9 marks)

(b) Given the heat equation

$$\frac{\partial u}{\partial t}=0.5\frac{\partial^2 u}{\partial x^2}, \qquad 0<x<1, \quad t>0,$$

with boundary conditions, $u(0,t)=20e^{-t}$ and $u(1,t)=60e^{-2t}$ for $t>0$ and initial condition $u(x,0)=20+40x$ for $0\le x\le 1$. By using implicit Crank-Nicolson method, solve the heat equation at first level only for $t\le 0.1$ by taking $\Delta x=h=0.25$, and $\Delta t=k=0.1$ using your calculator.

(11 marks)

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Formulae

Nonlinear equations

Secant method : $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$, $i = 0,1,2,\ldots$

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0,1,2,\ldots$

System of linear equations

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$

Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Cubic spline:

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k)$$

$$\left.\begin{array}{l} h_k = x_{k+1} - x_k \\ d_k = \dfrac{f_{k+1} - f_k}{h_k} \end{array}\right\}, k = 0,1,2,\ldots,n-2$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,\ldots,n-2$$

Natural cubic spline :

$m_0 = 0$

$m_n = 0$

$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, \quad k = 0,1,2,\ldots,n-2$

Numerical differentiation and integration

Differentiation:

First derivatives:

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x) - f(x-h)}{h}$

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3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Integration:

$\frac{1}{3}$ Simpson's rule: $\int_a^b f(x)dx \approx \dfrac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Gauss quadrature: For $\int_a^b f(x)dx$, $x = \dfrac{(b-a)t+(b+a)}{2}$

2-points: $\int_{-1}^{1} f(x)dx \approx g\left(-\dfrac{1}{\sqrt{3}}\right) + g\left(\dfrac{1}{\sqrt{3}}\right)$

Eigen value

Power Method : $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,.....$

Shifted Power Method: $\mathbf{A}_{\text{shifted}} = \mathbf{A} - s\mathbf{I}$, $\lambda_{\text{smallest}} = \lambda_{\text{Shifted}} + s$

Ordinary differential equations

Initial value problems:

Euler's method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

Boundary value problems:

Finite difference method:

$y_i' \approx \dfrac{y_{i+1} - y_{i-1}}{2h}$ $\qquad y_i'' \approx \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Partial differential equations

Heat equation- Implicit Crank-Nicolson:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \right)$$

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Wave equation- Finite difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Poisson equation-Finite difference method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = f_{i,j}$$

Finite element method

Heat flow problem in 1 dimension for $p \le x \le q$

$$N(x) = [N_1(x) \ \ N_2(x) \ \ \cdots \ \ N_n(x)]$$

$N_m(x) = \lfloor N_m^e(x) \ \rfloor$ is global shaped function for element e at node m

$$\mathbf{T} = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}$$, is the temperature vector at node

$$\mathbf{KT} = \mathbf{F}_b - \mathbf{F}_L$$

where

stiffness matrix, $\mathbf{K} = \int_p^q \mathbf{B}^T Ak\mathbf{B}\, dx$ or

$K_{ij} = \int_p^q A(x)k(x)\frac{dN_i}{dx}\frac{dN_j}{dx}\, dx$ is a square matrix with dimension $n \times n$,

boundary vector, $\mathbf{F}_b = \left[N_i A(x)k(x)\frac{dT}{dx} \right]_p^q$ have the dimension $n \times 1$,

load vector, $\mathbf{F}_L = -\int_p^q \mathbf{N}_i Q(x)\, dx$ have the dimension $n \times 1$.