UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	ENGINEERING MATHEMATICS III
CODE	:	BSM 2913
COURSE	:	2 BDD / BDI / BEE / BEI / BFF / BFI
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

PART A

Q1 (a) Given the force field

$$\mathbf{F}(x,y,z) = e^{y}\,\mathbf{i} + xe^{y}\mathbf{j} + 0\mathbf{k}.$$

(i) Prove that $\mathbf{F}$ is conservative.
(ii) Find its scalar potential function $\phi(x,y,z)$ such that $\mathbf{F} = \nabla\phi$.
(iii) Hence, calculate the amount of work done by $\mathbf{F}(x,y,z)$ in moving a particle from the point $(1,0,0)$ to $(-1,0,0)$.

(12 marks)

(b) Use Green's Theorem to evaluate the line integral $\int_C (x+y)dx + (3y+y^2-x^2)dy$, where C is the triangle with vertices $(0,0)$, $(2,0)$ and $(2,4)$ oriented in a counter clock-clockwise direction.

(8 marks)

Q2 (a) Use Divergence Theorem to find the outward flux

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\, dS$$

of the vector field $\mathbf{F}(x,y,z) = (x^3 - e^y)\mathbf{i} + (y^3 + \sin z)\mathbf{j} + (z^3 - xy)\mathbf{k}$, across the surface σ of the region that is enclosed by the hemisphere $z = \sqrt{4-x^2-y^2}$ and the $xy-$plane.

[Hint: Use spherical coordinates.]

(10 marks)

(b) Use Stokes' Theorem to evaluate

$$\iint_\sigma (\nabla\times\mathbf{F})\cdot\mathbf{n}\, dS,$$

where $\mathbf{F}(x,y,z) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$ and σ is the portion of the paraboloid $z = 4 - x^2 - y^2$ above the $xy-$plane with $\mathbf{n}$ the outward unit normal vector field to σ.

(10 marks)

PART B

Q3 (a) Show that the function $f(x,t)=\sin(n\pi x)\cos(n\pi ct)$ satisfies the wave equation $c^2\dfrac{\partial^2 f}{\partial x^2}=\dfrac{\partial^2 f}{\partial t^2}$ for any positive integer n and any constant c.

(5 marks)

(b) Given the function $f(x,\theta)=x^2e^{rx\theta}$. Find $\dfrac{\partial^2 f}{\partial x\partial\theta}$ where r is a constant.

(2 marks)

(c) Given $z=\ln(1+xy)$. By using total differential, approximate $\ln(1+(-0.09)(1.98))$ as (x,y) moves from the point $(0,2)$ to the point $(-0.09,1.98)$.

(4 marks)

(d) Find the local extrema and saddle point(s) of $f(x,y)=4xy-(x^4+y^4)$.

(9 marks)

Q4 (a) Given $\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}}\dfrac{x^2y}{x^2+y^2}\,dy\,dx$. Evaluate the integral by using polar coordinates.

(5 marks)

(b) Find the mass of the solid bounded by the cylinder $x^2+y^2=49$ and the plane $z=6$ in the first octant with the density function

$$\delta(x,y,z)=\frac{4}{(1+x^2+y^2)^3}.$$

(6 marks)

(d) Let G be the solid in the first octant bounded by the two spheres $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=1$. Sketch the region G and then by using spherical coordinates, evaluate

$$\iiint_G \frac{z}{x^2+y^2+z^2}\,dV.$$

(9 marks)

Q5 (a) Sketch the graph for the following vector-valued functions.

(i) $\mathbf{r}(t)=(3+2t)\,\mathbf{i}+(5-3t)\,\mathbf{j}+(2-4t)\,\mathbf{k},\quad t\in\Re.$

(ii) $\mathbf{r}(t)=\dfrac{1}{2}t\,\mathbf{i}+\cos 3t\,\mathbf{j}+\sin 3t\,\mathbf{k},\quad 0\le t\le 2\pi.$

(4 marks)

(b) Find the vector equation of the line which is tangent to the curve

$$\mathbf{r}(t) = t^2\,\mathbf{i} - \frac{1}{t+1}\mathbf{j} + (4 - t^2)\,\mathbf{k}, \quad t \neq -1$$

at the point (4, 1, 0).

(4 marks)

(c) Suppose that a particle moves along a circular helix such that its position vector at time t is

$$\mathbf{r}(t) = (4\cos \pi t)\mathbf{i} + (4\sin \pi t)\mathbf{j} + t\mathbf{k}.$$

Find its speed when $t = 2$ and the distance travelled of the particle during the time interval $1 \leq t \leq 5$.

(3 marks)

(d) Find the curvature $\kappa(t)$ for the curves

(i) $\mathbf{r}(t) = t\mathbf{i} + at^2\mathbf{j} + 0\mathbf{k}.$

(ii) $\mathbf{r}(t) = a\sin t\,\mathbf{i} + a(1 - \cos t)\mathbf{j} + 0\mathbf{k}.$

(iii) If both the curves coincide and have the same curvature when $t = 0$, find the value of a.

(9 marks)

Q6 (a) Find a unit vector in the direction in which the function

$$f(x, y) = \frac{1}{x} + \frac{1}{y}$$

increases most rapidly at $P(-1, 1, 0)$ and find the rate of change of f in that direction.

(5 marks)

(b) Evaluate the following line integral

$$\int_C x^2 dx + xy dy + z^2 dz,$$

where C is a parametric curve defined by $C: x = \sin t,\ y = \cos t,\ z = t^2,\ 0 \leq t \leq \frac{\pi}{2}.$

(5 marks)

(c) Find the centroid of the surface σ, where σ is the portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

$$\left[\text{Hint: Centroid of a surface } \sigma \text{ is defined by} \quad \bar{x} = \frac{\iint_\sigma x\, dS}{\text{area of } \sigma},\ \bar{y} = \frac{\iint_\sigma y\, dS}{\text{area of } \sigma},\ \bar{z} = \frac{\iint_\sigma z\, dS}{\text{area of } \sigma}.\right]$$

(10 marks)

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Formulae

Polar coordinates: $x = r\cos\theta,\ y = r\sin\theta$ and $x^2 + y^2 = r^2$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$$

Cylindrical coordinates: $x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $x^2 + y^2 = r^2$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Spherical coordinates: $x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi,\ \rho^2 = x^2 + y^2 + z^2,$
$0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

The directional derivatives, $D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$; The **gradient** of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then

The **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$.

The **unit tangent vector**, $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **principal unit normal vector**, $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

Curvature, $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

Green Theorem:

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Gauss Theorem:

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \nabla\cdot\mathbf{F}\,dV$$

Stokes' Theorem:

$$\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS = \int_C \mathbf{F}\cdot d\mathbf{r}$$

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Arc Length of Plane Curve and Space Curve

For a plane curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ on an interval $[a, b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} \, dt.$$

For a space curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ on an interval $[a, b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt.$$