KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

PEPERIKSAAN AKHIR
SEMESTER I
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NAMA MATA PELAJARAN	: MATEMATIK III
KOD MATA PELAJARAN	: DSM 2913
KURSUS	: 2 DFA, 2 DFX, 2 DFY, 2 DDM, 2 DDX, 2 DDT
TARIKH PEPERIKSAAN	: NOVEMBER 2006
JANGKA MASA	: 3 JAM
ARAHAN	: JAWAB **SEMUA** SOALAN DARIPADA **BAHAGIAN A** DAN **TIGA (3)** SOALAN DARIPADA **BAHAGIAN B**.

KERTAS SOALAN INI MENGANDUNGI 6 MUKA SURAT

PART A

Q1 (a) Find either $F(s)$ or $f(t)$ as indicated.

(i) $\mathcal{L}\left\{5t^3+4t\right\}$.

(ii) $\mathcal{L}\left\{t\cos t\right\}$.

(iii) $\mathcal{L}^{-1}\left\{\dfrac{5}{s+5}\right\}$.

(iv) $\mathcal{L}^{-1}\left\{\dfrac{10}{(s+2)^2+25}\right\}$.

(15 marks)

(b) Given $f(t)=\begin{cases} t, & 0\le t<3, \\ 6-t, & 3\le t<6, \\ 0, & t\ge 6. \end{cases}$

(i) Write $f(t)$ in unit step function form.

(ii) Find $\mathcal{L}\left\{f(t)\right\}$.

(5 marks)

Q2 Solve the differential equation below by using Laplace transform.

(a) $y'+4y=e^{-4t}, \quad y(0)=2$.

(9 marks)

(b) $y''-6y'+8y=0, \quad y(0)=0, \quad y'(0)=-3$.

(11 marks)

PART B

Q3 (a) Given matrices $A=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 6\\ 2 & 6 & 13\end{bmatrix}$, $B=\begin{bmatrix}3 & -8 & 3\\ -1 & 7 & -3\\ 0 & -2 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2 & -1\\ 5 & -4\\ -1 & 0\end{bmatrix}$.

(i) Find $C^T A$.

(ii) Find $|B|$.

(iii) Find AB and give your conclusion.

(iv) From the conclusion given in (a)(iii), solve the system of linear equation below by using the inverse method.

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 6z &= 3 \\ 2x + 6y + 13z &= 5 \end{aligned}$$

(10 marks)

(b) Consider the system of linear equation,

$$\begin{aligned} x + 2y + z &= 10 \\ -x - 2y - 2z &= 1 \\ -x - y \quad\quad &= 4 \end{aligned}$$

(i) Write the system in matrix form, $AX = B$.

(ii) Write the augmented matrix $[A|B]$.

(iii) Do the row operations given accordingly on $[A|B]$. Then, solve the system by using the Gauss-Jordan elimination method.

$$R_1 \to R_2 + R_1$$
$$R_3 \to R_3 + R_1$$
$$R_2 \leftrightarrow R_3$$
$$R_2 \to R_2 + R_3$$
$$R_3 \to (-1)R_3$$
$$R_1 \to R_1 - 2R_2$$
$$R_1 \to R_1 - R_3$$

(10 marks)

Q4 (a) Given the vectors $\mathbf{u} = 3\mathbf{i} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{j}$. Find

(i) $3\mathbf{w} - (\mathbf{v} - \mathbf{u})$.
(ii) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$
(iii) a vector unit with the same direction as $\mathbf{v}$.

(8 marks)

(b) Find an equation of a plane which passes through points $P(3, 2, 1)$ and parallel to a plane with equation $3x - 3y - 5z + 1 = 0$.

(7 marks)

(c) Find an equation of the line that passes through points $A(2, -1, 3)$ and $B(-1, 1, -2)$.

(5 marks)

Q5 (a) Given $z_1 = 3 - 2i$, $z_2 = 4i$ and $z_3 = 1 - 4i$. Find

(i) $\overline{z_1 + 3z_2}$. (ii) $\dfrac{z_2}{z_3}$. (iii) $(z_2)^2(z_1)$.

(8 marks)

(b) Given $z_4 = 2 + 3i$.

(i) Write z_4 in polar form.
(i) Find $(z_4)^4$.
(ii) Find $(z_4)^{4/3}$ and write your answer in $a + ib$ form, in three decimal places.

(12 marks)

Q6 Solve the following differential equation.

(a) $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$.

(10 marks)

(b) $y'' - 5y' + 6y = -e^x$, by using the variation of parameters method.

(10 marks)

FORMULAS

Table 1 : Characteristic Equation and General Solution

Differential equation: $ay'' + by' + c = 0$. Characteristic equation : $am^2 + bm + c = 0$.		
Case	**roots m_1 and m_2**	**General solution**
1	$m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2$	$y = (A + Bx)e^{mx}$
3	$m_1 = \alpha + \beta i$ $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2 : Undetermined Coefficient Method $ay'' + by' + c = f(x)$

$F(x)$	$y_p(x)$
a, where a is a constant	$x^r c$
$P_n(x) = (B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0)$	$x^r (B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r (P\cos\beta x + Q\sin\beta x)$

Notes : r is the smallest non negative integer, $\{r = 0, 1 \text{ or } 2\}$ to ensure no alike terms between $y_p(x)$ and $y_c(x)$.

$$y = y_c(x) + y_p(x).$$

Table 3 : Variation of Parameters method $ay'' + by' + c = f(x)$

The general solution of variation of parameters method is $y = y_c + y_p$ where $y_p = uy_1 + vy_2$. with $u = -\int \frac{y_2 f(x)}{W} dx$, $v = \int \frac{y_1 f(x)}{W} dx$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$