KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

PEPERIKSAAN AKHIR
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NAMA MATA PELAJARAN	:	MATEMATIK KEJURUTERAAN IV
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TARIKH PEPERIKSAAN	:	NOVEMBER 2006
JANGKA MASA	:	3 JAM
ARAHAN	:	JAWAB **SEMUA** SOALAN DARI BAHAGIAN A DAN **TIGA (3)** SOALAN DARI BAHAGIAN B. LAKUKAN SEMUA PENGIRAAN DALAM 3 TEMPAT PERPULUHAN.

KERTAS SOALAN INI MENGANDUNGI 9 MUKA SURAT

PART A

Q1 (a) Given the boundary value problem

$$y''(t) = \frac{2t}{1+t^2} y'(t) - \frac{2}{1+t^2} y(t) + 1, \qquad 0 \le t \le 0.8,$$

with conditions $y(0) = 1.25$ and $y(0.8) = 1.189$. By using finite difference method with $\Delta t = h = 0.2$, find the value of $y(0.2)$, $y(0.4)$ and $y(0.6)$.

(10 marks)

(b) Given the Laplace's equation,

$$u_{xx} + u_{yy} = 0, \qquad 0 < x < 1 \text{ and } 0 < y < 1$$

with the boundary conditions $u(0, y) = y^2$ and $u(1, y) = y$ for $0 \le y < 1$, $u(x, 0) = x^2$ and $u(x, 1) = (x-1)^2$ for $0 \le x \le 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta y = 0.5$, solve the Laplace's equations by using finite difference method. (Do **NOT** solve the system of linear equations)

(10 marks)

Q2

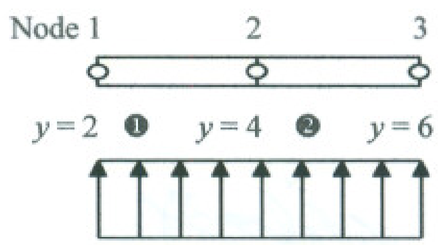

Figure Q2

Consider the heat flow equation $\frac{d}{dy}\left(A\,k\,\frac{dT}{dy}\right) + Q(y) = 0$, for $2 \le y \le 6$, on a fin consisting of three nodes and two elements, as shown in figure Q2.

Given the following values,

The cross-sectional area, $A = 20\ \text{m}^2$

The thermal conductivity, $k = 4\ \text{J}/^{\text{o}}\text{C m s}$

The heat supply per unit time and per unit length, $Q = 50\ \text{J/s m}$

The boundary condition, $T_1 = T\ |_{y=2} = 0\,^{\text{o}}\text{C}$.
The flux, $q_3 = q\,|_{y=6} = 5\ \text{J / m}^2\ \text{s}$
$T(y)$ is the temperature at length y.

The shape functions are defined as below;

$$N_1(y) = \begin{cases} N_1^1(y) = 2 - \dfrac{1}{2}y, & \text{for } y \text{ in element 1,} \\ 0, & \text{otherwise.} \end{cases}$$

$$N_2(y) = \begin{cases} N_2^1(y) = y - 2, & \text{for } y \text{ in element 1,} \\ N_2^2(y) = 3 - \dfrac{1}{2}y, & \text{for } y \text{ in element 2,} \\ 0, & \text{otherwise.} \end{cases}$$

$$N_3(y) = \begin{cases} N_3^2(y) = \dfrac{1}{2}y - 2, & \text{for } y \text{ in element 2,} \\ 0, & \text{otherwise.} \end{cases}$$

By using Galerkin method for approximation on the linear model, $T = \alpha_1 + \alpha_2 y$,

(a) prove that the temperature at each nodal point, $T_2 = T\ |_{y=4} = 0.625\,^{\text{o}}\text{C}$ and $T_3 = T\ |_{y=6} = -0.625\,^{\text{o}}\text{C}$,

(18 marks)

(b) find the flux at the left end of the fin, $q_1 = q\ \big|_{y=2}$.

(2 marks)

PART B

Q3 (a) (i) Find the positive root for $f(x) = e^x - 3x^2$ by using bisection method from the Figure Q3 below with $|b-a| = 1$ and $c_0 = 3.5$. Do your calculation until 4 iterations.

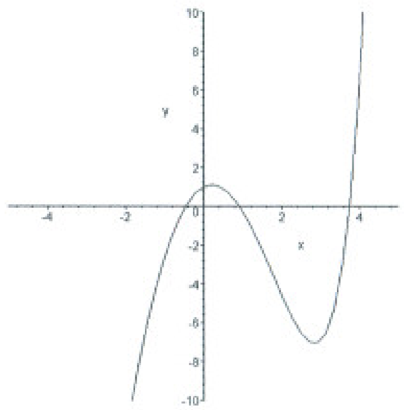

Figure Q3

(ii) For bisection method in part a(i), what can you say when $|f(c_i)| > \varepsilon$ where $c_i = \dfrac{a_i + b_i}{2}$?

(10 marks)

(b) Given the system of linear equations, $\boldsymbol{AX} = \boldsymbol{B}$ as follow,

$$3x_1 + x_2 + 2x_3 = 12$$
$$x_1 + 2x_2 + 2x_3 = 11$$
$$2x_1 + 2x_2 + 3x_3 = 2$$

(i) State 4 rules to show that the matrix $\boldsymbol{A}$ is symmetric positive definite.

(ii) Hence, solve the above system of linear equations using Cholesky method.

(10 marks)

Q4 Given the set data of sine functions in Table Q4.

Table Q4

i	0	1	2	3
x_i	$\pi/9$	$\pi/6$	$\pi/3$	$\pi/2$
$f(x_i)=\sin x_i$	0.342	0.500	0.866	1.000

(a) Find the approximate value of $f(\pi/5)$ by using Newton's divided difference method.

(7 marks)

(b) Find the approximate value of $f(\pi/5)$ by using natural cubic spline.

(13 marks)

Q5 (a) The following Table Q5 gives the distance, x of an object at various time, t.

Table Q5

Time, t (sec)	2.7	2.8	2.9	3.0	3.1	3.2
Distance, x (m)	11.2	11.7	12.3	13.1	14.0	15.0

Find the acceleration, $a=\dfrac{d^2x}{dt^2}$ of the object at $t=3.0$ second using 3-point and 5-point difference formulas with a suitable value of step size, h for both formulas.

(6 marks)

(b) (i) Find the approximate value for $\int_0^3 e^{x^2}\,dx$ by using 1/3 Simpson's rule. The interval [0, 3] is used with 6 subintervals.

(ii) Find the approximate value for $\int_1^4 \sqrt{x^2+3}\,dx$ by using 3-point Gauss quadrature.

(14 marks)

Q6 (a) Below are matrix A and its tabulated calculations (Table Q6) using the power method, for finding the largest eigenvalue and its corresponding eigenvector, with $\varepsilon = 0.005$.

Table Q6

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

k	$v^{(k)}$			$[Av^{(k)}]^T$			m_{k+1}
0	0	0	1	1	2	3	3
1	0.333	0.667	1	2	4	4.667	4.667
2	0.429	0.857	1	2.286	4.571	5.142	5.142
3	0.444	0.889	1	2.334	4.667	5.223	5.223
4	0.447	0.894	1	2.34	4.681	5.234	5.234
5	0.447	0.894	1	2.341	4.683	5.236	5.236
6	0.447	0.894	1				

(i) Use the shifted power method to get its smallest eigenvalue with its corresponding eigenvector.

(ii) Verify your answers in part (i).

(9 marks)

(b) Given the initial value problem

$$y' - xy = x \ ; \quad y(0) = 1$$

with $h = 0.2$ at $0 \le x \le 1$. (The exact solution is $y = 2e^{\frac{x^2}{2}} - 1$)

(i) Find the approximation solutions and its absolute errors for the above problem using Modified Euler method.

(ii) Find the approximation solutions and its absolute errors for the above problem using Improved Euler method.

(iii) For this above problem, give the best method

(11 marks)

FORMULAS

Nonlinear Equations

Bisection method

If $f(a_i)f(c_i) < 0$ then $a_{i+1} = a_i, b_{i+1} = c_i$ where $c_i = \dfrac{a_i + b_i}{2}$

System of Linear Equations

Cholesky Factorization: $\boldsymbol{A = LU}$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & \vdots \\ l_{31} & l_{32} & l_{33} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} l_{11} & l_{12} & l_{13} & \cdots & l_{n1} \\ 0 & l_{22} & l_{23} & \cdots & l_{n2} \\ 0 & 0 & l_{33} & \cdots & l_{n3} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & l_{nn} \end{pmatrix}$$

Interpolation

Newton divided difference method:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \cdots + f_0^{[n]}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Cubic Spline:

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k)$$

$$k = 0, 1, 2, 3, \ldots, n-1$$

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, \quad k = 0, 1, 2, 3, \ldots, n-1 \ ,$$

$$b_k = 6(d_{k+1} - d_k), \quad k = 0, 1, 2, 3, \ldots, n-2,$$

Natural Cubic Spline :

$$m_0 = 0,$$

$$m_n = 0,$$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, \qquad k = 0, 1, 2, 3, \ldots, n-2,$$

Numerical Differentiation and Integration

First derivatives :

3 point central difference: $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

5 point difference: $f'(x) = \dfrac{1}{12h}[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)]$

Second Derivatives :
3 point central difference :

$$f''(x) = \frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)]$$

5 point difference:

$$f''(x) = \frac{1}{12h^2}[-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)]$$

Simpson $\frac{1}{3}$ rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4(f_1 + f_3 + f_5 + \cdots f_{n-5} + f_{n-3} + f_{n-1}) + 2(f_2 + f_4 + \cdots + f_{n-4} + f_{n-2})\right]$$

Gauss Quadrature:

for $\int_a^b f(x)dx, \quad x = \frac{(b-a)t + (b+a)}{2}$

3 points $\int_{-1}^{1} f(x)dx \approx \frac{5}{9}f(-\sqrt{\frac{3}{5}}) + \frac{8}{9}f(0) + \frac{5}{9}f(\sqrt{\frac{3}{5}})$

Eigen value

$$|A - \lambda I| = 0$$

Shifted Power Method

$$A_{shifted} = A - sI$$

$$v^{(k+1)} = \frac{1}{m_{k+1}} A_{shifted} v^{(k)}, \qquad k = 0, 1, 2, \ldots..$$

Ordinary Differential Equation

Initial Value Problem:

Modified Euler Method $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$ where $k_1 = hf(x_i, y_i)$

$k_2 = hf(x_i + h, y_i + k_1)$

Improved Euler method or Mid point Method $y_{i+1} = y_i + k_2$ where $k_1 = hf(x_i, y_i)$

$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

Boundary Value Problem:

Finite Difference Method

$$y'_i \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y''_i \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial Differential Equation

Laplace Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = 0 \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Finite Element

The element of stiffness matrix, $\boldsymbol{K}$;

$$\boldsymbol{K}_{ij} = \int_a^b \frac{dN_i}{dx} Ak \frac{dN_j}{dx}\, dx$$

The component of boundary vector, $\boldsymbol{f}_b$;

$$\boldsymbol{f}_b = -\left[N^T Aq(x)\right]_a^b = -\begin{pmatrix} [N_1 Aq]_a^b \\ [N_2 Aq]_a^b \\ \vdots \\ [N_n Aq]_a^b \end{pmatrix}$$

The component of load vector, $\boldsymbol{f}_L$;

$$\boldsymbol{f}_L = \int_a^b N^T Q\, dx = \begin{pmatrix} \int_a^b N_1 Q\, dx \\ \int_a^b N_2 Q\, dx \\ \vdots \\ \int_a^b N_n Q\, dx \end{pmatrix}$$

$$\boldsymbol{K}\boldsymbol{a} = \boldsymbol{f}_b + \boldsymbol{f}_L$$