KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

PEPERIKSAAN AKHIR SEMESTER I SESI 2006/2007

NAMA MATA PELAJARAN	:	MATEMATIK III
KOD MATA PELAJARAN	:	DSM 2933
KURSUS	:	2DEE, 2DET, 2DEX
TARIKH PEPERIKSAAN	:	NOVEMBER 2006
JANGKA MASA	:	3 JAM
ARAHAN	:	JAWAB **SEMUA** SOALAN DALAM BAHAGIAN **A** DAN **TIGA** (**3**) SOALAN PILIHAN DALAM BAHAGIAN **B**.

KERTAS SOALAN INI MENGANDUNGI 6 MUKA SURAT

PART A

Q1 (a) Solve

$$x\frac{dy}{dx}=1-y+xy\tan x.$$

(8 marks)

(b) By using undetermined coefficients method, solve

$$y''-4y=3x+e^{2x}$$

with $y(0)=0$ and $y'(0)=1$.

(12 marks)

Q2 (a) Obtain the particular solution for first order differential equation

$$\frac{\sin x}{1+y}\frac{dy}{dx}=\cos x$$

subject to initial condition $y\left(\frac{\pi}{2}\right)=1$.

(8 marks)

(b) Solve

$$y''-y'-2y=e^{3x}$$

with using the variation of parameter.

(12 marks)

PART B

Q3 (a) Find

$$\lim_{x \to 0^+} \tan x \ln x .$$

(10 marks)

(b) If $1200\, cm^2$ of a material is available to make a box with square base and an open top, find the largest volume of the box.

(10 marks)

Q4 Evaluate

(a) $\int x^2 \sin 3x \, dx$.

(8 marks)

(b) $\int (4x+6)\sqrt{x^2+3x} \, dx$

(4 marks)

(c) $\int \frac{3x^2-2x+12}{(x^2+3)(x+2)} \, dx$.

(8 marks)

Q5 (a) Evaluate

$$\int_1^3 \left(4x^2 - x\right) dx$$

(i) using the trapezoidal rule with $n = 4$.

(ii) without using the trapezoidal rule.

Hence, calculate the absolute error of approximation.

(8 marks)

(b) Find the area of the region bounded above by $y = 9 - x^2$, bounded below by $y = x + 1$, and bounded on the sides by $x = -1$ and $x = 2$.

(4 marks)

(c) The arc of the curve $y = \sqrt[3]{x}$ from $(1,1)$ to $(8,2)$ is rotated about the y-axis. Find the area of the resulting surface.

(8 marks)

Q6 (a) Solve the homogeneous equation $(xe^{\frac{y}{x}} + y)dx - xdy = 0$.

(8 marks)

(b) Given $[e^y - \sin(x-y) + x^2]dx + [xe^y + \sin(x-y) - 3y^2]dy = 0$.

(i) Show that the given equation is an exact equation.

(4 marks)

(ii) Then, solve it.

(8 marks)

Table 1. Differentiation And Integration Formula

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$

Table 2. Arc Length and Surface Area of Revolution

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad S = 2\pi \int_a^b f(x)\sqrt{1 + \left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \qquad S = 2\pi \int_a^b g(y)\sqrt{1 + \left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Table 3. Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$; Characteristic equation : $am^2 + bm + c = 0$		
Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 4. Particular Integral of $ay''+by'+cy = f(x)$

$f(x)$	$y_k(x)$
$P_n(x) = A_0 + A_1x + \cdots + A_nx^n$	$x^r(B_0 + B_1x + \cdots + B_nx^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_k(x)$ corresponds to the complementary function $y_h(x)$.

Table 5. Variation of Parameters Method for $ay''+by'+cy = f(x)$

$y(x) = uy_1 + vy_2$	
$u = -\int \frac{y_2 f(x)}{aW}dx + A$	$v = \int \frac{y_1 f(x)}{aW}dx + B$
$W = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$	