CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME	:	1DAA / 1DAM / 1DAE / 1DAU/ 1DAT
EXAMINATION DATE	:	DECEMBER 2015/ JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

CONFIDENTIAL

SECTION A

Q1 (a) The vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are given by $\boldsymbol{p} = 4\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{q} = -\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$. Find

(i) $|2\boldsymbol{p} - 3\boldsymbol{q}|$

(ii) $\boldsymbol{p} \times \boldsymbol{q}$

(iii) The angle between vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

(9 marks)

(b) Given the coordinate of plane $A\,(1, 3, -1)$, $B\,(-1, -1, 2)$ and $C\,(-2, 0, 4)$. Find

(i) The equation of a line that passes through A and B.

(ii) The equation of a plane containing A, B and C.

(11 marks)

Q2 (a) Given $z = 1+2i$. Express in the form $a + bi$, the complex number $\dfrac{(2 - 3z^2)}{z^2}$.

(5 marks)

(b) Given $z_1 = 5 + i$ and $z_2 = 2 - 3i$. If $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2}$, find z

(6 marks)

(c) Simplify $\left(\dfrac{1-2i}{1+3i}\right)^2 + \left(\dfrac{1+i}{1-2i}\right)$ in the form of $a+bi$.

(9 marks)

SECTION B

Q3 (a) Solve the following exponential equation $9^{\frac{x}{3}} - \dfrac{27}{3^{-x}} = 0$.

(6 marks)

(b) Find the value of x without using calculator $\log_2\left(\sqrt{64}\right)^x = \log_4 32$.

(8 marks)

(c) Simplify $\sqrt[5]{32x^5}\,\sqrt[3]{16x^8}$.

(6 marks)

Q4 (a) Find the root of the equation $f(x) = 2\sin x + x^2 - 2$ in the interval $[0,1]$ accurate to within $\varepsilon = 0.005$ using secant method. Show all your calculation in three decimal places.

(7 marks)

(b) Express $\dfrac{2x^2 + 4x - 9}{x^3 + 2x^2 - 5x - 6}$ in partial fraction.

(7 marks)

(c) Expand $\dfrac{1}{\left(\sqrt{1+x}\right)}$ in ascending powers of x until the term involving x^3.

(6 marks)

Q5 (a) Find the sum of $\sum_{k=1}^{9} \left(3k^2 - 12k + 8\right)$.

(4 marks)

(b) The 3rd and 10th term of an arithmetic sequence are 28 and 49 respectively. Find

(i) The first term and the common difference.

(ii) The sum of the first 45 terms.

(8 marks)

(c) The 3rd term of a geometric series is $\dfrac{1}{12}$ and the 5th term is $\dfrac{1}{432}$. Find

(i) The first term and the common ratio where the common ratio, $r > 0$.

(ii) The sum of the first 9 terms.

(8 marks)

Q6 (a) By using double angle formula, simplify $\sin 15^\circ \cos 15^\circ$.

(6 marks)

(b) By using the half-angle formula, find the value of $\cos 15^\circ$.

(6 marks)

(c) Given $\sqrt{10}\sin\theta + \sqrt{6}\cos\theta = r\sin(\theta + \alpha)$ and $0^\circ \le \theta \le 360^\circ$.

(i) Find r and α

(3 marks)

(ii) Thus, find the value of θ if $\sqrt{10}\sin\theta + \sqrt{6}\cos\theta = 3$.

(5 marks)

CONFIDENTIAL

Q7. (a) Given $\mathbf{A} = 2\begin{bmatrix} 1 & x & 5 \\ y & 4 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 & -x \\ 0 & 3 \\ z & z \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 2 & 5 \\ 6 & -3 \end{bmatrix}$

(i) Solve x, y and z if $\mathbf{AB} = \mathbf{C}$

(5 marks)

(ii) Find $(\mathbf{AB})^{\mathrm{T}}$

(4 marks)

(b) Given

$$2x + y + z = 3$$
$$-3x - 2y = -7$$
$$3x + y - z = 6$$

(i) Write the matrix equation $\mathbf{AX} = \mathbf{B}$ of the above system of equation.

(1 mark)

(ii) Find the determinant of matrix **A**.

(2 marks)

(iii) Solve the above system for x, y and z by using Gauss-Jordan elimination method. Do this following operation in order:

$R_2 + R_3$,

$R_3 - R_1$,

$R_3 \Leftrightarrow R_1$,

$R_3 - 2R_1$,

$-R_2$,

$R_3 - R_2$,

$\frac{R_3}{4}$,

$R_1 + 2R_3$,

$R_2 - R_3$.

(8 marks)

- END OF QUESTION –

CONFIDENTIAL

FINAL EXAMINATION

SEMESTER / SESSION : SEM 1 / 2015/2016 PROGRAMME: 1 DAA / 1 DAE / 1 DAM / 1 DAT / 1 DAU

COURSE : ALGEBRA COURSE CODE: DAS 10103

Formulae

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}, r > 1 \text{ or } S_n = \frac{a(1 - r^n)}{1 - r}, r < 1, \quad S_\infty = \frac{a}{1 - r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM 1 / 2015/2016 PROGRAMME: 1 DAA / 1 DAE / 1 DAM / 1 DAT / 1 DAU

COURSE : ALGEBRA COURSE CODE: DAS 10103

$$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$$

and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Vector

$$\cos\theta = \frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} \quad \text{or} \quad \boldsymbol{a}\bullet\boldsymbol{b} = x_1x_2 + y_1y_2 + z_1z_2, \quad \boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$|a| = \sqrt{x^2 + y^2 + z^2}$,

$x = x_0 + a_1 t, \quad y = y_0 + a_2 t, \quad z = z_0 + a_3 t$ and $\dfrac{x - x_o}{a_1} = \dfrac{y - y_o}{a_2} = \dfrac{z - z_o}{a_3}$

$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$

Complex number

$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \dfrac{y}{x}$