UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2DAA / 3 DAA / 3DAM
EXAMINATION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTIONS	:	A) ANSWER **ALL** QUESTIONS IN **PART A** B) ANSWER **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 (a) Find the inverse Laplace for these expressions

(i) $\dfrac{8}{s^3}+\dfrac{2}{s}$

(ii) $\dfrac{2s+5}{(s+3)^2}$

(10 marks)

(b) Find $L^{-1}\left\{\dfrac{5s+1}{s^2-s-12}\right\}$

(10 marks)

Q2 Solve the given initial and boundary value problem of differential equation using Laplace transform.

(a) $y''-6y'+8y=0$; Initial value problem : $y(0)=0,\quad y'(0)=-3$

(8 marks)

(b) $y''-7y'+12y=2$; Boundary value problem : $y=1,\ y'=5,$ when $t=0$.

(12 marks)

PART B

Q3 (a) Given ordinary differential equation $\dfrac{dy}{dx}=\dfrac{x+3y}{2x}$.

(i) Show that the ODE is a homogeneous equation.

(ii) Thus, solve the equation.

(10 marks)

(b) Given $\frac{dy}{dx}+\frac{y}{2x}=\frac{1}{y}$.

(i) Show that the differential equation above is an exact equation.

(ii) Then, solve the equation.

(10 marks)

Q4 During the semester break, you work at the Parit Raja factory. You need to remove a metal with its core temperature of 1300 °F from a furnace and placed the metal on a table in a room that had a constant temperature of 75°F. One and half hour after it is, removed the core temperature is measured as 1020°F, when you check the temperature of the metal. The temperature of the metal must be below 550 °F before you can transfer it to the next section. You removed the metal at 8.00 am and your lunch start at 1.30 pm.

(a) Find the rate of change of the temperature dT/dt in term of T and T_s, given the temperature of the metal $T(t)$ and the ambient temperature T_s

(4 marks)

(b) Show that $T-T_s=Ae^{-kt}$.

(4 marks)

(c) Using the observed initial temperatures of the metal, $T(0)$ = 1300, find the constant A. Hence find $T(t)$.

(4 marks)

(d) Using the observed temperatures of the metal, given T(1.5) = 1020, find the constant k.

(4 marks)

(e) If you removed the metal at 8.00 am, explain whether you would be able to go for your lunch at 1.30 pm.

(4 marks)

Q5 Solve the given second order differential equation by using the stated method.

(a) $y'' - 3y' + 2y = 6e^{2x}$ by method of undetermined coefficient.

(10 marks)

(b) $y'' - 5y' + 6y = -e^{x}$ by method of variations of parameters.

(10 marks)

Q6 (a) Find laplace transform for these functions:

(i) $f(t) = (t + t^2 + \frac{1}{6}t^3)e^t$

(ii) $f(t) = t^2 \sin 2t$

(10 marks)

(b) Consider the function

$$f(t) = \begin{cases} 1-t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

(i) Write the function $f(t)$ in the form of Heaviside / Unit-Step Function.

(ii) Find the Laplace transform of $f(t)$.

(10 marks)

Q7 During the semester break, your brother got married. You are in charge of boiling eggs. The eggs' temperature must be below 19°C before you can put the eggs in beautiful small baskets. You boiled until the temperature of the eggs is at 97°C . Then you put the eggs under running 17°C water to cool. After 8 minutes, the eggs' temperature is found to be 38°C. How much longer would you need to wait?

(a) Find the rate of change of the temperature dT/dt in term of T and T_s, given the temperature of the metal $T(t)$ and the ambient temperature T_s

(4 marks)

(b) Show that $T - T_s = Ae^{-kt}$.

(4 marks)

(c) Using the observed initial temperatures of the metal, $T(0) = 97$, find the constant A. Hence find $T(t)$.

(4 marks)

(d) Using the observed temperatures of the metal, given $T(8) = 38$, find the constant k.

(4 marks)

(e) Hence, how long before closing time should the soup be ready so that you could put it in the fridge and leave on time?

(4 marks)

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Formula

Table 1 : Laplace transform.

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt = F(s)$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t),\ n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$f(t-(a)\ H(t-(a)$	$e^{-as} F(s)$

Table 2 : Initial and Boundary Value Problem

If $L\{y(t)\} = Y(s)$ then

$L\{y'(t)\} = sY(s) - y(0)$

$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

Table 3 : Indefinite differentiation

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\left(\frac{dt}{dx}\right)$
$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$
$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sec u = \sec u\tan u\cdot\frac{du}{dx}$
$\frac{d}{dx}\csc u = -\csc u\cot u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cos u = -\sin u\cdot\frac{du}{dx}$

Table 4 : Indefinite integral

$\int \text{sek}^2\, x\, dx = \tan x + c$

$\int \text{kosek}^2\, x\, dx = -\text{kot}\, x + c$

$\int \text{sek}\, x \tan x dx = \text{sek}\, x + c$

$\int \text{kosek}\, x\, \text{kot}\, x dx = -\text{kosek}\, x + c$

$\int \text{sek}\, x\, dx = \ln\left|\ \text{sek}\, x + \tan x\ \right| + c$

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Table 5 : Solution of particular solution $ay'' + by' + cy = f(x)$

$f(x)$	$y_k(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C \cos\beta x$ atau $C \sin \beta x$	$x^r (p \text{ kos } \beta x + q \sin \beta x)$
$P_n(x) e^{\alpha x}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}$
$P_n(x)\begin{cases} kos \beta x & atau \\ \sin \beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, sin\, \beta x$
$Ce^{\alpha x}\begin{cases} kos \beta x & atau \\ \sin \beta x \end{cases}$	$x^r e^{\alpha x} (p\, kos\, \beta x + q\, sin\, \beta x)$
$P_n(x) e^{\alpha x}\begin{cases} kos \beta x & atau \\ \sin \beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}\, sin\, \beta x$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_k(x)$ and $y_h(x)$.

Table 6 : Variation of parameters method.

The general solution of $ay'' + by' + cy = f(x)$ variation of parameters method is $y(x) = uy_1 + vy_2$

with $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$,

$$u = -\int \frac{y_2 f(x)}{aW}\, dx + A,$$

$$v = \int \frac{y_1 f(x)}{aW}\, dx + B \text{ and}$$

the general solution $y = u_1 y_1 + u_2 y_2$