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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME	:	3 DAA / 3 DAE / 3 DAT
EXAMINATION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTIONS	:	A) ANSWER **ALL** QUESTIONS B) ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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SECTION A

Q1 (a) Given $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. Find

(i) $\mathbf{a} - 3\mathbf{b}$

(ii) $\mathbf{a} \bullet \mathbf{b}$

(iii) $\mathbf{b} \times \mathbf{a}$

(iv) $|\, \mathbf{a} - 3\mathbf{b} \,|$

(10 marks)

(b) Find the parametric and symmetric equations of a line that passes through points P (4, – 2, 6) and Q (– 3, 3, – 6).

(5 marks)

(c) Find the equation of a plane containing (1, 1, 3) and normal to the vector $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

(5 marks)

Q2 (a) If $z_1 = 2 + 3i$ and $z_2 = 7 - 6i$. Determine

(i) $z_1 + z_2$

(ii) $z_1 - z_2$

(iii) $z_1 z_2$

(6 marks)

(b) Express $z = \dfrac{1+2i}{3+i}$ in

(i) standard form
(ii) polar form

(6 marks)

(c) Find all the third roots of $z = 3 + 4i$ by using De Moivre theorem.

(8 marks)

SECTION B

Q3 (a) (i) Simplify $\left(\dfrac{x^{-3}y^{2}z^{-1}}{x^{-4}y^{9}z^{2}}\right)^{2}$

(4 marks)

(ii) Solve for x, given $25^{x-1} = 125^{4x}$

(4 marks)

(b) Given log 2 = 0.3010 and log 3 = 0.4771. Find

(i) log 12

(ii) $\log_2 64$

(iii) $\log_2 64 - \log_2 16$

(6 marks)

(c) Given $x = 2 + \sqrt{5}$. Find

(i) x^2

(ii) $x(2 + x)$

(6 marks)

Q4 (a) Solve for x, given $2x^2 - 6x - 11 = 0$.

(6 marks)

(b) Express $\dfrac{x-18}{x(x-3)}$ as a sum of partial fractions with constant numerators.

(7 marks)

(c) Given $\dfrac{(x+3)(x+1)}{x-1} \leq 0$

By using sign analysis, solve the above inequality.

(7 marks)

Q5 (a) Given $\cos\theta = \dfrac{5}{13}$. By using double-angle formula, find

(i) $\sin 2\theta$

(ii) $\cos 2\theta$

(iii) $\tan 2\theta$

(7 marks)

(b) By using half-angle formula, find the exact value of

(i) $\sin 45^\circ$

(ii) $\cos 120^\circ$

(iii) $\tan 135^\circ$

(6 marks)

(c) Find all values of θ if given $2\sin^2\theta - \sin\theta = 1$ for $0 \le \theta \le 360^\circ$.

(7 marks)

Q6 (a) Let $A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \\ -1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 3 & -2 \\ 6 & -5 & 4 \end{pmatrix}$ $C = \begin{pmatrix} 4 & 2 & -1 \\ 9 & 0 & 1 \\ 4 & 5 & -2 \end{pmatrix}$ $D = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 5 & 9 \\ 1 & 1 & -2 \end{pmatrix}$

Find the value of

(i) $C - D$

(ii) $A^{T} - 2B$

(iii) $A^{T}C$

(iv) CD

(10 marks)

(b) Given a system of equations :

$$2x - y + 4z = -3$$
$$x - 2y - 10z = -6$$
$$3x + 4z = 7$$

(i) Write the system into matrix equation, AX = B

(ii) Write the augmented matrix, [A | B]

(iii) Do the row operations one after another.

$R1 \longleftrightarrow R2$

$R2 - 2R1 \longrightarrow R2$

$R3 - 3R1 \longrightarrow R3$

$\frac{1}{3}R2$

$R3 - 6R2 \longrightarrow R3$

(iv) Continue the row operations from (iii) and find x, y and z.

(10 marks)

-END OF QUESTIONS-

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Formulae

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

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Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = \left(c_{ij}\right)^T \qquad A^{-1} = \frac{1}{|A|} Adj(A)$$

Vector

$$\cos\theta = \frac{\boldsymbol{a} \bullet \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} \quad \text{or} \quad \cos\theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \quad \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$|\mathrm{a}| = \sqrt{\mathrm{a}_1^2 + \mathrm{a}_2^2 + \mathrm{a}_3^2}, \; x = x_0 + a_1 t, \quad y = y_0 + a_2 t, \quad z = z_0 + a_3 t \text{ and } \frac{x - x_o}{a_1} = \frac{y - y_o}{a_2} = \frac{z - z_o}{a_3}$$

Complex number

If $z = re^{i\theta}$, then $z^n = r^n e^{in\theta}$

If $z = re^{i\theta}$, then $z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta + 2k\pi}{n}\right)i}$.

If $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

If $z = r(\cos\theta + i\sin\theta)$ then $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{(\theta + 2k\pi)}{n} + i\sin\frac{(\theta + 2k\pi)}{n}\right)$