Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30602 / BEE 31602
PROGRAMME	:	1 BEJ / 2 BEJ / 2 BEV / 3 BEJ / 3 BEV / 4 BEJ / 4 BEV
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Given the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, \quad t > 0,$$

with the boundary conditions, $u(0,t) = 20t^2$ and $u(1,t) = 10t$ for $t > 0$ and the initial condition, $u(x,0) = x(1-x)$ for $0 \le x \le 1$. By using the explicit finite-difference method, solve the heat equation by taking $\Delta x = h = 0.25$ and $\Delta t = k = 0.01$ until $t = 0.02$.

(10 marks)

(b) The air pressure $p(x,t)$ in an organ pipe is governed by the wave equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}, \quad 0 < x < l, \quad 0 < t$$

where l is the length of the pipe and c is a physical constant. When the pipe is open, the boundary conditions are

$$p(0,t) = p_0 \text{ and } p(l,t) = p_0, \quad 0 < t.$$

Assume that the length, l and physical constant, c are 1 respectively and the initial conditions are

$$p(x,0) = p_0 \cos 2\pi x \text{ and } \frac{\partial p}{\partial t}(x,0) = 0, \quad 0 \le x \le 1.$$

Use finite-difference method to approximate the pressure for an open pipe with $p_0 = 0.9$ for $t = 0.1$, 0.2 and 0.3. Use $\Delta x = 0.25$ and $\Delta t = 0.1$.

(15 marks)

Q2 (a) Let $f(x) = x^4 - 18x^2 + 45$.

(i) Verify that $f(x) = 0$ has a root in the interval $[1, 2]$.

(2 marks)

(ii) Hence, find the root of $f(x) = 0$ by using secant method and iterate until $|f(x_i)| < 0.005$.

(7 marks)

(iii) Given that the exact value of the root is $x = \sqrt{3}$. Compute the absolute error in the approximation in **Q2**(a)(ii).
[Hint: Absolute error $= |\text{exact value} - \text{approximate value}|$]

(2 marks)

(b) Consider the following vectors

$$\vec{A} = p\underset{\sim}{i} + q\underset{\sim}{j},\ \vec{B} = -2\underset{\sim}{i} + \underset{\sim}{j} \text{ and } \vec{C} = \underset{\sim}{i} + 3\underset{\sim}{j},$$

where $\vec{A}$ is an unknown vector.

Given that

$$(\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C}) = (2p - 3q)\underset{\sim}{i} + 14\underset{\sim}{k}.$$

Find the vector $\vec{A}$ by using Gauss elimination method.

[Hint: $\vec{a} \times \vec{b} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = (a_1 b_2 - a_2 b_1)\underset{\sim}{k}$]

(14 marks)

Q3 (a) Bessel function often arises in advanced engineering analyses such as in the study of an electric field. **Table Q3(a)** represents selected values for the Bessel function.

Table Q3(a): Values for Bessel function

x	1.8	2.0	2.2	2.4
$J_1(x)$	0.581	0.576	0.556	0.520

Approximate $J_1(1.9)$ and $J_1(2.3)$ by using the third-degree Newton's divided difference method.

(8 marks)

(b) The values of an unknown function, $f(x)$ at several points are given in **Table Q3(b)**.

Table Q3(b): Values of function, $f(x)$

x	2.1	2.3	2.5	2.7	2.9
$f(x)$	5.375	5.801	6.211	6.367	6.914

Find $f'(2.6)$ and $f''(2.3)$ by using 3-point central difference formula with suitable step length.

(7 marks)

(c) A curve $y = f(x)$ between $x = a$ and $x = b$ is rotated about the x-axis. The volume of the solid of revolution, thus generated, is given by the following integral

$$\int_a^b \pi y^2 \, dx .$$

Calculate the volume of the solid of revolution when the curve $y = e^x$ between $x = 0$ and $x = 0.5$ is rotated about the x-axis by using appropriate Simpson's rule. Consider $\pi = 3.142$ with four sub-intervals.

(10 marks)

Q4 (a) Solve the initial-value problem defined by

$$y^2y' + x^2y' = y^2 - x^2, \qquad y(0) = 1$$

at $x = 0\,(0.2)\,0.4$ by using the fourth-order Runge-Kutta method.

(10 marks)

(b) A thin rod of length, l is moving in the *xy*-plane. The rod is fixed with a pin on one end and a mass at the other end. This system is represented in the form of boundary-value problem (BVP) as follows

$$\theta''(t) - \frac{g}{l}\theta(t) = 0, \quad \text{for} \quad 0 \le t \le 0.4,$$

where boundary conditions are $\theta(0) = 0$ and $\theta(0.4) = 1$. The parameter values are given as $g =$ gravitational force (9.81 m/s^2) and $l = 0.9$ m. Approximate the angle θ (in radian) for $h = 0.1$ by using finite-difference method.

(15 marks)

- END OF QUESTION -

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FORMULA

Nonlinear equations

Secant method: $$x_{i+2}=\frac{x_i f(x_{i+1})-x_{i+1}f(x_i)}{f(x_{i+1})-f(x_i)}, \quad i=0,1,2,\ldots$$

Interpolation

Newton's divided difference method:

$$P_n(x)=f_0^{[0]}+f_0^{[1]}(x-x_0)+f_0^{[2]}(x-x_0)(x-x_1)+\ldots+f_0^{[n]}(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$

Numerical differentiation and integration

Differentiation

First derivatives:

3-point central difference: $$f'(x)\approx\frac{f(x+h)-f(x-h)}{2h}$$

Second derivatives:

3-point central difference: $$f''(x)\approx\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$$

Integration:

Simpson's ⅓ rule: $$\int_a^b f(x)dx\approx\frac{h}{3}\left[f_0+f_n+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1}f_i+2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2}f_i\right]$$

Simpson's ⅜ rule:

$$\int_a^b f(x)dx\approx\frac{3}{8}h\left[f_0+f_n+3(f_1+f_2+f_4+f_5+\ldots+f_{n-2}+f_{n-1})+2(f_3+f_6+\ldots+f_{n-6}+f_{n-3})\right]$$

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Ordinary differential equations

Initial value problems:

Fourth-order Runge-Kutta method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$, $k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite-difference method: $y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$, $y_i'' \approx \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$

Partial differential equation

Heat equation: Explicit finite-difference method:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Wave equation: Finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$