UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	APPLIED STOCHASTICS MODEL
COURSE CODE	:	BWB 22303
PROGRAMME	:	2 BWB
EXAMINATION DATE	:	JUNE 2015/JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) Zarf can take a course in computers or in chemistry. If Zarf take the computer course, then she will receive an A grade with probability 1/2, while if she takes the chemistry course then she will receive an A grade with probability 1/3. Zarf decides to base her decision on the flip of fair coin. What is the probability that Zarf will get an A in chemistry?

(4 marks)

(b) Suppose that our experiment consists of tossing two fairs coins. Letting Y denote the number of heads appearing, the Y is a random variable taking on one of the values 0, 1, 2. Proof that the probability of tossing two fairs coins equal to 1.

(6 marks)

(c) Let X denote the number of additional years that a randomly chosen 75 years old person will live. If the person has high blood pressure, denoted as event H, then X is a geometric (p=0.3) random variable. Otherwise, if the person's blood pressure is regular, denoted as event R, then X has a geometric distribution with parameter (p=0.01).

X- Random variable for the number of additional years that a randomly chosen 75 years old person will live.

H- The conditioning event of a person who has high blood pressure that make X~G (p=0.3).

R- The conditioning event of a person who has regular blood pressure that make ~ $G \sim$ (p=0.01).

$$P(X = x \mid H) = \begin{cases} 0.3(0.9)^{x-1}, x = 1, 2..., \\ 0 \qquad ,otherwise \end{cases}$$

$$P(X = x \mid R) = \begin{cases} 0.01(0.99)^{x-1}, x = 1, 2,.... \\ 0 \qquad ,otherwise \end{cases}$$

If 40% of all seventy years olds have high blood pressure, what is the PMFs of X?

(10 marks)

Q2 (a) Customer arrived in a certain store open at 9 p.m. according a Poisson process with $\lambda = 4$ per hour.

(i) What is the probability that exactly one customers has arrived by 9.30pm?

(5 marks)

(ii) What is the probability that total of five customers have arrived by 11.30pm.

(5 marks)

(iii) What is the probability that total of ten has arrived by the time the stores closes in between 9.30 to 11.30?

(5 marks)

(b) X is said to have no memory, or memoryless, property if for all $s,t \geq 0$. Thus proof $P(X>s+t|X>t)=P(X>s)$.

(5 marks)

Q3 (a) The Chapman- Kolmogorov equations provide a method for computing the n step transition probabilities. These equations are $P_{ij}^{n+m}=\sum_{k=0}^{\infty}P_{ik}^{n}P_{kj}^{m}$ for all $n,m\geq 0$, all i,j. Proof that the equation $P_{ij}^{n+m}=\sum_{k=0}^{\infty}P_{ik}^{n}P_{kj}^{m}$.

(6 marks)

(b) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if has rained for past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rain yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2

(i) Represent a four state Markov chain that the process is in state 0 if it rained both today and yesterday, state 1 if it rained today but not yesterday, state if it rained yesterday but not today, state 3 if it did not rain either yesterday or today.

(4 marks)

(ii) Using MC in b(i) , calculate the transition matrix P^2

(6 marks)

(iii) From your answer b(i), given that it rained on Monday and Tuesday what is the probability that it will rain on Thursday.

(4 marks)

Q4 (a) A man walks along a four-block stretch of Park Avenue. If he is at corner 1, 2, or 3, then he walks to the left or right with equal probability. He continues until he reaches corner 4, which is a bar, or corner 0, which is his home. If he reaches either home or the bar, he stays there. Determine which state are absorbing, recurrent and transient.

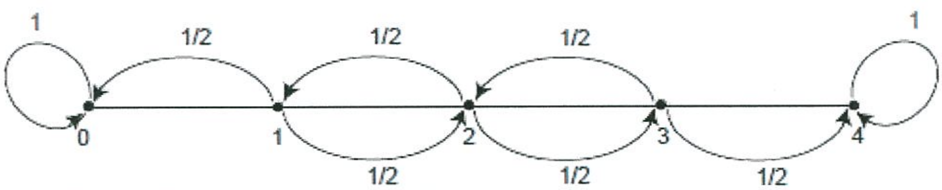

Figure Q4(a) :A man walks along a four-block stretch of Park Avenue

(4 marks)

(b) (i) Birth-Death processes and queuing system is a Markov chain if the transition rates satisfy $q_{ij} = 0$ for $|1-j|>1$. A single server queue (M/M/1) is a customer's arrive at a single server queuing station according to $Poisson(\lambda)$.The service times of successive customers are iid $\exp(\mu)$ random variables. The service is offered by first come first served (FCFS) basis. The size of waiting room is infinite. Let *P(t)* be the number of customers in the system (the number of customers in the queue and being served) at time *t*. Assume $\lambda = 5/\sec$ and $\mu = 8/\sec$.Find the numbers of customer at P_0, P_1, P_2 and the percentage for each P_0, P_1, P_2.

(6 marks)

(ii) Proof that $E(X) = \sum_{i=0}^{\infty} ipi$ .Hint $\left[\sum_{i=0}^{\infty} i(\alpha)^{i+1} = \frac{1}{(1-\alpha)^2} \right]$

(4 marks)

(c) Suppose that a one-celled organism can be in one of two states either *A* or *B*. An individual in state *A* will change to state *B* at an exponential rate *α*; an individual in state *B* divides into two new individuals of type *A* at an exponential rate *β*. Define an appropriate continuous-time Markov chain for a population of such organisms and determine the appropriate parameters for this model.

(6 marks)

Q5 (a) The Brownian motion process, sometimes called the Wiener process, is one of the most useful stochastic processes in applied probability theory. State variations on Brownian Motion.

(4 marks)

(b) A stochastic process $\{X(t), t \geq 0\}$ is said to be a Brownian motion process. Define the Brownian motion process.

(6 marks)

(c) Prove that;

$$E\left[X(t) \mid X(\mu), 0 \leq \mu \leq s\right] = X(s)E\left[e^{Y(t)-Y(s)}\right].$$

(5 marks)

(d) Let *X*(*t*) be the price of FMC stock at time *t* years from the present. Assume that *X*(*t*) is a geometric Brownian motion with drift $\mu = -0.05$/year and volatility $\sigma =$ 0.4 / $yr^{1/2}$. If the current price of FMC stock is RM 2.50. What is the probability that the price will be at least RM2.60 one year from now?

(5 marks)

- END OF QUESTION -