Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME	:	2 DAU
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A.** B) ANSWER **THREE (3)** QUESTIONS ONLY IN **SECTION B.**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

SECTION A

Q1 (a) Evaluate the following integral by using substitution.

$$\int \frac{2x-8}{(x^2-8x+3)^3}\,dx$$

(5 marks)

(b) Determine whether the following integrations are improper or proper integral. Give your reason.

(i) $\int_{-1}^{2} \frac{3}{x^2-1}\,dx$

(2 marks)

(ii) $\int_{-\infty}^{\infty} e^{\sqrt{x}}\,dx$

(2 marks)

(iii) $\int_{1}^{2} \frac{(x^4-1)^2}{x^2}\,dx$

(2 marks)

(c) Solve the following integral by using Simpson's rule, using $h = 0.125$. Write the answer to 3 decimal places.

$$\int_{0}^{1} \sqrt{\frac{x}{1+x}}\,dx$$

(9 marks)

Q2 (a) Determine the area of the region bounded by the curve $y = 2 + x - x^2$ and line $y + x + 1 = 0$.

(7 marks)

(b) (i) Sketch the curve $y^2 = 8x$ and $y = x^2$.

(3 marks)

(ii) Determine the points of intersection.

(5 marks)

(iii) Find the volume of the solid generation when the region bounded is revolves 360° about the x-axis.

(5 marks)

SECTION B

Q3 (a) Given $f(x) = 3x^2 - kx + 7$. Find the value of k if

(i) $f(2) = 11$.

(3 marks)

(ii) $f(0) = 3k - 8$.

(4 marks)

(b) Given $g(x) = 4x^2 - 5x - 6$.

(i) Find $g(15)$.

(3 marks)

(ii) Find x when $g(x) = 0$.

(4 marks)

(c) The function h is defined by $h : x \rightarrow 5x - 3$. The function k is such that $h \circ k : x \rightarrow \frac{3}{x^2 - 2}$. Find the function k.

(6 marks)

Q4 (a) Let $f(x) = \begin{cases} -2 & x < 0 \\ x + 3 & 0 \leq x \leq 2 \\ x^2 & x > 2 \end{cases}$. Find

(i) $f(-10)$, $f(2)$ and $f(5)$.

(3 marks)

(ii) $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.

(2 marks)

(iii) $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$.

(2 marks)

(iv) $\lim_{x \to 1} f(x)$ and $\lim_{x \to 3} f(x)$.

(2 marks)

(b) Referring to **Q4**(a), check whether $f(x)$ continues at $x = 0$ and $x = 2$.

(5 marks)

(c) Given $\lim_{x \to 1} g(x) = 10$ and $\lim_{x \to 1} h(x) = 12$. Calculate

(i) $\frac{1}{2} \lim_{x \to 1} g(x) + 3 \lim_{x \to 1} h(x)$.

(2 marks)

(ii) $\lim_{x \to 1} \left(\frac{2g(x) - h(x)}{g(x).h(x)} \right)^2$.

(4 marks)

Q5 (a) Differentiate with respect to x using the chain rule.

$$y = \frac{1}{x^3 - 2x + 5}$$

(8 marks)

(b) Find the implicit differentiation for

$$3y^2 - 2x^2 = 2xy$$

(5 marks)

(c) A piece of 2.5 meter of wire is cut to form a circle and a square. Calculate the radius, r of the circle and the length, x of a square so that the total area is minimum.

[*Hint:* Area of circle is $A = \pi r^2$, Perimeter of circle is $p = 2\pi r$]

(7 marks)

Q6 (a) Evaluate $\int 3x \cos x \, dx$ by using integration by parts.

(4 marks)

(b) Determine the arc length of $x = \frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \leq y \leq 4$.

(6 marks)

(c) **Figure Q6(c)** shows the region bounded from the three lines: $x = y$, $y = -1$ and $y = 4 - x$.

(i) Find the point of intersection A, B, and C.

(6 marks)

(ii) Show that the area, A of the bounded region R is 9 unit^2.

(4 marks)

Q7 (a) Given two functions, $f(x) = 2x^2 + 3$ and $g(x) = 2 - x - x^2$.

(i) Calculate $f(x) - g(x)$.

(2 marks)

(ii) Calculate $f(x).g(x)$

(2 marks)

(iii) Find inverse function, $f^{-1}(x)$.

(3 marks)

(iv) Find the composite function, $f \circ g\,(x)$.

(3 marks)

(b) Calculate the following limits.

(i) $\lim_{x\to 2} \frac{2x-3}{x^2+3x-4}$

(2 marks)

(ii) $\lim_{x\to 2} \frac{2x-4}{x^2-4}$

(2 marks)

(iii) $\lim_{x\to 1} \frac{2-\sqrt{x+3}}{x-1}$

(2 marks)

(iv) $\lim_{x\to \infty} \frac{2x-7}{x^3+x-4}$

(2 marks)

(v) $\lim_{x\to 0} \frac{2cosx-2}{x^3}$

(2 marks)

- END OF QUESTIONS -

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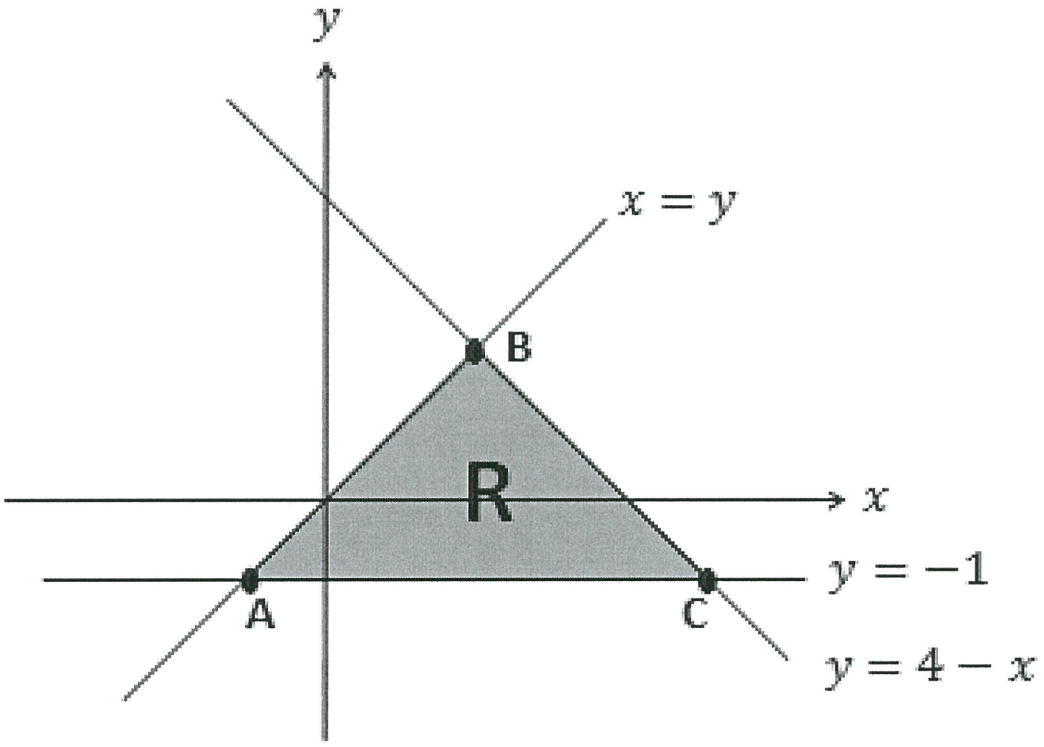

Figure Q6(c)

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FORMULAE

Differentiations

$\frac{d}{dx}(ax^n) = nax^{n-1}$

$\frac{d}{dx}(u^n) = nu^{n-1}.\frac{du}{dx}$

$\frac{d}{dx}\left(\frac{1}{\sqrt{u}}\right) = \frac{1}{2\sqrt{u}}.\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u.\frac{du}{dx}$

$\frac{d}{dx}(\ln u) = \frac{1}{u}.\frac{du}{dx}$

$\frac{d}{dx}(\mathrm{k}u) = k.\frac{du}{dx}$

$\frac{d}{dx}(\sin u) = \cos\ u.\frac{du}{dx}$

$\frac{d}{dx}(\cos u) = -\sin\ u.\frac{du}{dx}$

$\frac{d}{dx}(\tan u) = sec^2\ u.\frac{du}{dx}$

$\frac{d}{dx}(\sec\ u) = \sec\ u.tan\ u.\frac{du}{dx}$

$\frac{d}{dx}(uv) = uv' + vu'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

Basic Integration

$\int kdx = kx + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int sinxdx = -cosx + C$

$\int cosxdx = sinx + C$

Integration By Parts

$\int udv = uv - \int vdu$

Arch Length

$\int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

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Area of bounded Region

$$\int_c^d [u(y) - v(y)]dy$$

Volume of solid generation

$$V = \pi \int_c^d [(y_2)^2 - (y_1)^2]\, dx$$